ON AN APPLICATION OF L. EHRENPREIS' METHOD TO ORDINARY DIFFERENTIAL EQUATIONS

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Introduction.

In 1956 Ehrenpreis [3] considered an application of the sheaf theory to differential equations and gave a criterion for the existence of global solutions of differential equations where the existence of local solutions are assured.

We shall apply this method to systems of ordinary and linear differential equations with coefficients meromorphic in a domain D on the plane C of one complex variable z.

Let $\mathfrak{O}$ and $\mathfrak{M}$ be the sheaves of all germs of functions holomorphic and meromorphic in D respectively. Let a_{jk} $(j, k=1, 2, \cdots, p)$ be functions meromorphic in D.

For any element $f=(f^1, f^2, \cdots, f^p)$ of $\mathfrak{M}^p$, we define

$$Tf=\left(\frac{df^1}{dz}+\sum_{k=1}^{p} a_{1k}f^k, \frac{df^2}{dz}+\sum_{k=1}^{p} a_{2k}, \cdots f^k, \frac{df^p}{dz}+\sum_{k=1}^{p} a_{pk}f^k\right).$$

Then T is a homomorphism of $\mathfrak{M}^p$ into itself.

Let $\mathfrak{A}$ be the sheaf of all germs $f\in\mathfrak{M}^p$ which satisfy the homogeneous equation $Tf=0$, and $T\mathfrak{M}^p$ be the sheaf of all germs $g\in\mathfrak{M}^p$ for each of which there exists $f\in\mathfrak{M}^p$ such that $g=Tf$.

Then we have the exact sequence $0\to\mathfrak{A}\to\mathfrak{M}^p\xrightarrow{T}T\mathfrak{M}^p\to 0$. Therefore we have also the exact sequence of cohomology groups

$$H^0(D, \mathfrak{A})\to H^0(D, \mathfrak{M}^p)\xrightarrow{T}H^0(D, T\mathfrak{M}^p)\to H^1(D, \mathfrak{A})\to H^1(D, \mathfrak{M}^p)\xrightarrow{T}H^1(D, T\mathfrak{M}^p)\to\cdots.$$

Since $H^1(D, \mathfrak{M}^p)=0$ by Theorem 1 of the present paper, we have $H^1(D, \mathfrak{A})=H^0(D, T\mathfrak{M}^p)/TH^0(D, \mathfrak{M}^p)$. Therefore $H^1(D, \mathfrak{A})$ measures how many $g\in H^0(D, \mathfrak{M}^p)$, which are locally of the form $g=Tf$ for any point of D, are not globally of the form $g=Tf$ for $f\in H^0(D, \mathfrak{M}^p)$.

Calculating the cohomology group $H^1(D, \mathfrak{A})$ we have the following theorem:

If $H^0(D, T\mathfrak{M}^p)=TH^0(D, \mathfrak{M}^p)$, then D is simply or doubly connected.

If D is simply connected, then the necessary and sufficient condition for $H^0(D, T\mathfrak{M}^p)=TH^0(D, \mathfrak{M}^p)$ is that there exist linearly independent solutions $f_1, f_2, \cdots$ and

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