ON EVANS' SOLUTION OF THE EQUATION $\Delta u = Pu$ ON RIEMANN SURFACES

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Introduction.

Let R be an open Riemann surface. By a density $P(z)$ on R we mean a non-negative continuously differentiable function of local parameters $z=x+\sqrt{-1}\,y$ such that the expression $P(z)dxdy$ is invariant under the change of local parameters z. Then we can consider the elliptic partial differential equation

$$(E) \qquad \Delta u(z) = P(z)u(z), \qquad \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2},$$

which is invariantly defined on R. Throughout this paper, we always assume

$$P(z)dxdy \not\equiv 0$$

on R. By a solution u of (E) on an open subset D of R we mean that u is twice continuously differentiable function satisfying (E) on D.[1] An *Evans' solution* $e(z)$ of (E) on R is a solution of (E) on R satisfying

$$\lim_{R \ni z \to A_\infty} e(z) = \infty,$$

where A_∞ is the Alexandroff's ideal boundary point of R. The purpose of this paper is to give a sufficient condition for the existence of Evans' solution of (E) on R.

Let $(R_n)_{n=0}^\infty$ be a normal exhaustion of R and $\Omega_{0,n}$ $(n>0)$ be the continuous function on $\bar{R}_n - R_0$ such that $\Omega_{0,n}$ is a solution of (E) on $R_n - \bar{R}_0$ with $\Omega_{0,n}=1$ on ∂R_0 and $\Omega_{0,n}=0$ on ∂R_n. Then there exists a continuous function Ω_0 on $R-R_0$ such that Ω_0 is a solution of (E) on $R-\bar{R}_0$ with $\Omega_0=1$ on ∂R_0 and

$$\Omega_0(z) = \lim_{n\to\infty} \Omega_{0,n}(z) > 0$$

on $R-R_0$. Clearly $\Omega_0(z)$ does not depend on the special choice of exhaustions $(R_n)_{n=1}^\infty$.

We consider the condition

$$(\Omega) \qquad \sigma = \inf_{z\in R-R_0} \Omega_0(z) > 0.$$

It is easy to see that the condition (Ω) does not depend on the special choice of

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1) For fundamental properties of solutions of (E), refer to the list in pp. 152–153 in [4].