ON A UNIQUENESS CONDITION FOR SOLUTIONS OF THE DIRICHLET PROBLEM CONCERNING A QUASI-LINEAR EQUATION OF ELLIPTIC TYPE

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§1. Introduction.

In the present paper, we are concerned with a uniqueness condition for solutions of the Dirichlet problem concerning a quasi-linear elliptic equation of the second order

$$\sum_{i,j=1}^{m} a_{ij}(x, \nabla u)\frac{\partial^2 u}{\partial x_i \partial x_j} = f(x, u, \nabla u).\text{[1]} \tag{1.1}$$

Recently, Kusano [1][2] has established the maximum principle for quasi-linear elliptic equations of the general form, and as its application, he has given a uniqueness condition for solutions of the equation (1.1). We will here show that the uniqueness of solutions may be established under a weaker condition, by the method adopted in author's previous note [2].

In this paper, x denotes a point $(x_1, x_2, \cdots, x_m)$ in the m-dimensional Euclidean space, and we use the notations $\partial_i u$ for $\partial u/\partial x_i$, and $\partial_i\partial_j u$ for $\partial^2 u/\partial x_i \partial x_j$. Furthermore we introduce a differential operator $L[v; u]$ of elliptic type by the expression

$$L[v; u] = \sum_{i,j=1}^{m} a_{ij}(x, \nabla v)\partial_i\partial_j u,$$

and then the equation (1.1) can be written as follows:

$$L[u; u] = f(x, u, \nabla u). \tag{1.2}$$

By a solution of the equation (1.2) in a domain D, we mean a function belonging to $C^2[D]$ and satisfying the equation (1.2) in D.

§2. Hypotheses on the functions $a_{ij}(x, p)$ and $f(x, u, p)$.

Let D be a bounded domain in the m-dimensional Euclidean space and let $\dot{D}$ be the boundary of D.

We define a domain $\mathfrak{D}_0$ in the $2m$-dimensional Euclidean space as follows:

$$\mathfrak{D}_0 = \{(x, p);\ x \in D,\ |p| < +\infty\},\text{[3]}$$

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1) ∇u denotes the vector $(\partial u/\partial x_1, \partial u/\partial x_2, \cdots, \partial u/\partial x_m)$.

2) The numbers in brackets refer to the list of references at the end of this paper.

3) The notation $|p|$ means $\{\sum_{i=1}^{m} |p_i|^2\}^{1/2}$, where $p = (p_1, \cdots, p_m)$.