ON THE EXISTENCE OF AN ESSENTIAL PICARD'S PERFECT SET

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1. Introduction.

Let E be a perfect set and D a complementary domain of E. If any meromorphic function in D with its singularities at each point of E admits at most n Picard's exceptional values at any neighborhood of each point of E, then E is said to be an n-Picard's perfect set. A 2-Picard's perfect set is simply said to be a Picard's perfect set.

Recently Matsumoto [5] proved the existence of n ($\geqq 3$)-Picard's perfect set E. Further he constructed a 3-Picard's perfect set E in any neighborhood of any point of which there is a meromorphic function with just 3 Picard's exceptional values. In his construction E is of zero capacity. At the same time Carleson [2] proved independently the existence of 3-Picard's perfect set E in a class $N_{\mathfrak{B}}$ but cap $E > 0$.

In the present paper we shall extend the notion of Picard's perfect set and prove the existence of a Picard's perfect set in a new sense. We shall make use of the standard notions of the Nevanlinna theory [6].

Hayman [3] developed the Nevanlinna theory in a great extent in a case of the unit disc. Our main idea is due to the nice theorems I and II in [3].

2. Definition of an essential Picard's perfect set.

Let $\mathfrak{L}(X)$ be a class of meromorphic functions which are Lindelöfian in a domain X in Heins' sense [4]. This is the same as a class of meromorphic functions of bounded type in X. Let E be a perfect set lying on a simple closed curve γ and D a complementary domain of E. Let D_1 and D_2 be two domains bounded by γ. Let $N(p)$ be a generic neighborhood of any generic point p of E. Let $\mathfrak{M}$ be a class of meromorphic functions in D with essential singularities on E.

If any element of f in $\mathfrak{M} - \mathfrak{L}(N(p) \frown D_1) - \mathfrak{L}(N(p) \frown D_2)$ has n-Picard's exceptional values at most in any $N(p)$ of each point p of E, then E is said to be an essential n-Picard's perfect set. If $n = 2$, then E is simply said to be an essential Picard's perfect set.

This modification of the definition of n-Picard's perfect set E brings us an advantage. In fact, if $E \notin N_{\mathfrak{B}}$, then there exists a bounded analytic function

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