ECKMANN-FRÖLICHER CONNEXIONS ON ALMOST ANALYTIC SUBMANIFOLDS

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1. Almost analytic submanifolds.

We consider a $2m$-dimensional differentiable manifold M_{2m} of class C^∞ and with an almost complex structure $F_j{}^h$:

$$F_j{}^i F_i{}^h = -A_j^h, \tag{1.1}$$

where A_j^h denotes a unit tensor and the indices $h, i, j, \cdots$ run over the range $1, 2, \cdots, 2m$. We call such a manifold an almost complex space.

It is well known that the condition for an almost complex structure to be induced from a complex structure is the vanishing of the Nijenhuis tensor [4]:

$$N_{ji}{}^h = F_j{}^l \partial_l F_i{}^h - F_i{}^l \partial_l F_j{}^h - (\partial_j F_i{}^l - \partial_i F_j{}^l) F_l{}^h, \tag{1.2}$$

where ∂_l denotes the partial differentiation with respect to the coordinates ξ^l. We call a complex space an almost complex space with vanishing Nijenhuis tensor.

We now consider a $2n$-dimensional submanifold M_{2n} $(2m > 2n)$:

$$\xi^h = \xi^h(\eta^a) \tag{1.3}$$

of class C^∞ where the indices $a, b, c, \cdots$ run over the range $1, 2, \cdots, 2n$. If the transform by $F_i{}^h$ of any vector tangent to M_{2n} is still tangent to M_{2n}, we call M_{2n} an almost analytic submanifold. A necessary and sufficient condition for M_{2n} to be almost analytic is

$$F_i{}^h B_b{}^i = {}'F_b{}^a B_a{}^h, \tag{1.4}$$

where ${}'F_b{}^a$ is a certain tensor of M_{2n} and

$$B_a{}^h = \partial_a \xi^h; \qquad \partial_a = \partial / \partial \eta^a.$$

From (1.1) and (1.4), we find

$$'F_c{}^b\, {}'F_b{}^a = -A_c^a. \tag{1.5}$$

Thus, we have [3]

THEOREM 1.1. *An almost analytic submanifold in an almost complex space is an almost complex space.*

On the other hand, by a straightforward computation, we have

$$B_c{}^j B_b{}^i N_{ji}{}^h = {}'N_{cb}{}^a B_a{}^h, \tag{1.6}$$

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