GENERAL CONNECTIONS $A\Gamma A$ AND THE PARALLELISM OF LEVI-CIVITA

BY TOMINOSUKE ŌTSUKI

In a previous paper [10], the author showed that for a normal general connection Γ[1] of an n-dimensional differentiable manifold $\mathfrak{X}$ we can define naturally two normal general connections $'\Gamma$ and $''\Gamma$ called the contravariant part and the covariant part of Γ respectively. In the present paper, the author will show that we can define products of a general connection and tensor fields of type (1, 1) on $\mathfrak{X}$ satisfying the associative law. According to this concept, $'\Gamma = Q\Gamma$ and $''\Gamma = \Gamma Q$, where Q is the inverse of P in the sense that $Q|P(T(\mathfrak{X})) = (P|P(T(\mathfrak{X})))^{-1}$ and $Q|P^{-1}(0) = P|P^{-1}(0)$ at each point of $\mathfrak{X}$. As an application, he will investigate a normal general connection $A\Gamma A$, where Γ is a metric regular general connection with respect to a metric tensor, A is a projection of $T(\mathfrak{X})$ and $A(T(\mathfrak{X}))$ and $A^{-1}(0)$ are invariant under $P = \lambda(\Gamma)$ respectively. Then, he will show that the well known parallelism of Levi-Civita in Riemannian geometry can be considered as a parallelism by means of a sort of general connections.

In this paper, the author will use the notations in [7], [8], [9], [10].

§1. Products of a general connection and tensor fields of type (1.1).

Let $\mathfrak{X}$ be a differentiable manifold of dimension n and Γ be a general connection of $\mathfrak{X}$ which is written in terms of local coordinates u^i as

(1.1) $$\Gamma = \partial u_j \otimes (P^j_i d^2u^i + \Gamma^j_{ih} du^i \otimes du^h)$$

or

(1.2) $$\Gamma = \partial u_j \otimes (d(P^j_i du^i) + \Lambda^j_{ih} du^i \otimes du^h),$$

where

(1.3) $$\Lambda^j_{ih} = \Gamma^j_{ih} - \frac{\partial P^j_i}{\partial u^h}.$$

For each coordinate neighborhood (U, u^i), we have two mappings

$$f_U\colon\ U \to \mathfrak{M}^2_n = \{(a^j_i, a^j_{ih})\}\ ^{2)}$$

by

(1.4) $$a^j_i \cdot f_U = P^j_i, \qquad a^j_{ih} \cdot f_U = \Gamma^j_{ih}$$

Received November 9, 1961.

1) See [8].

2) See [10], §2 or [7], §1.