A NOTE ON THE EXISTENCE OF SOLUTIONS OF DIFFERENCE-DIFFERENTIAL EQUATIONS

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Introduction. In [2] and [3], the author has discussed the existence of bounded and periodic solutions of a difference-differential equation such that

$$\frac{dx(t+1)}{dt} = ax(t+1) + bx(t) + f(x(t+1),\, x(t),\, t), \tag{0.1}$$

where a and b are constant, corresponding respectively to the cases where $f(0, 0, t)$ is bounded and periodic in t. His discussion proceeded there was essentially based on the assumption that every root of the characteristic equation $e^s(s-a)-b=0$ lies to the left of the straight line $\Re s = -\delta$, where δ is a positive constant.

The purpose of this note is to discuss the existence of solutions, not necessary to be bounded, of (0.1) under the condition for the roots of the characteristic equation weaker than that stated above, that is, the condition that some roots of $e^s(s-a)-b=0$ lie to the right of the imaginary axis. However, the assumptions upon $f(x, y, t)$ may be made strong.

1. Kernel functions. In (0.1), we suppose that every real part of all the roots of the characteristic equation $e^s(s-a)-b=0$ is less than δ (>0). Applying for (0.1) a transformation $e^{2\delta t}y(t+1) = x(t+1)$, (0.1) is transformed into an equation

$$\frac{dy(t+1)}{dt} = (a-2\delta)y(t+1) + be^{-2\delta}y(t) + e^{-2\delta t}f(e^{2\delta t}y(t+1),\, e^{2\delta(t-1)}y(t),\, t). \tag{1.1}$$

Then, we find that every real part of all the roots of the characteristic equation

$$e^s(s-a+2\delta) - be^{-2\delta} = 0 \tag{1.2}$$

corresponding to the linear equation

$$\frac{dy(t+1)}{dt} = (a-2\delta)y(t+1) + be^{-2\delta}y(t) \tag{1.3}$$

is less than $-\delta$.

Now, we shall define a kernel function for (1.3). Let $K_y(t)$ be a solution of (1.3) for $0 \leqq t < \infty$ under the initial conditions $K_y(t) = 0$ $(-1 \leqq t < 0)$ and $K_y(0) = 1$. Then, we call $K_y(t)$ the *kernel function* of (1.3) and it is useful to summarize the results concerning $K_y(t)$ which will be used later:

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