ON GENERALIZED UNISERIAL ALGEBRAS OVER A PERFECT FIELD

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Let A be a ring with a unit element satisfying the minimum condition; let N be the radical of A. We call A a generalized uniserial ring if every indecomposable left [right] ideal of A possesses only one composition series. A generalized uniserial algebra over a field F is defined similarly. Recently H. Kupisch [3] discussed such rings and proved that a (two-sided) indecomposable generalized uniserial algebra over an algebraically closed field is completely determined up to isomorphism by a certain system of invariants. In the present note we shall generalize his method to the case of algebras over a perfect field, starting from the fact that the residue class algebra $\bar{A}=A/N$ of a (two-sided) indecomposable generalized uniserial algebra A over a field F (modulo the radical N) has the structure $B\times_F D$, where B is a split semisimple algebra over F and D is a division algebra over F.

NOTATIONS. Let

$$A=\sum_{\kappa=1}^{k}\sum_{i=1}^{f(\kappa)}Ae_{\kappa,i}=\sum_{\kappa=1}^{k}\sum_{i=1}^{f(\kappa)}e_{\kappa,i}A$$

be a decomposition of A into direct sum of indecomposable left [resp. right] ideals; $e_{\kappa,i}$ $(1\leqq\kappa\leqq k,\ 1\leqq i\leqq f(\kappa))$ are mutually orthogonal primitive idempotents; $Ae_{\kappa,i}\cong Ae_{\lambda,j}$ if and only if $\kappa=\lambda$; $e_\kappa=e_{\kappa,1}$, $E_\kappa=\sum_i e_{\kappa,i}$, and $E=\sum_\kappa E_\kappa$ is the unit element of A. $c_{\kappa,ij}$ $(1\leqq\kappa\leqq k,\ 1\leqq i,\ j\leqq f(\kappa))$ be a system of elements of A such that $c_{\kappa,ii}=e_{\kappa,i}$, $c_{\kappa,ij}c_{\kappa,kl}=\delta_{jk}c_{\kappa,il}$; $g(A)=k$ be the number of simple constituents of $\bar{A}=A/N$. $V=V^{(0)}\supset V^{(1)}\supset\cdots\supset V^{(d)}=0$ be the upper Loewy series of an A-left module V; here $V^{(m)}=N^mV$. $V=V_{(d)}\supset\cdots\supset V_{(1)}\supset V_{(0)}=0$ be the lower Loewy series of V; here $V_{(m)}=\{v\mid v\in V,\ N^mv=0\}$. $d(V)=d$ be the length of the upper and lower Loewy series of V; $d(A)=\rho$ is the index of N, i. e. $N^{\rho-1}\neq 0$, $N^\rho=0$.

1. A certain system of generators of composition factor modules of a two-sided composition series of a generalized uniserial ring.

Let A be a generalized uniserial ring and let N be its radical. We first consider an (A, A) composition series of A, which is a refinement of the series $A\supset N\supset N^2\supset\cdots\supset N^\rho=0$:

(1) $$A=\mathfrak{z}_0^0\supset\mathfrak{z}_1^0\supset\cdots\supset\mathfrak{z}_{r_0}^0=N=\mathfrak{z}_0^1\supset\cdots\supset\mathfrak{z}_{r_1}^1=N^2=\mathfrak{z}_0^2\supset\cdots\supset\mathfrak{z}_{r_{\rho-1}}^{\rho-1}=N^\rho=0.$$

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