§ 1. Introduction.

The problem discussed in this paper is the one being concerned indirectly with queuing theory, but it happens during the study of the method used in the other two articles on the theory [4, 5].

Let $X_i$ ($i = 1, 2, \cdots$) be a sequence of mutually independent real valued random variables with the common distribution. Put $S_n = \sum_{i=1}^{n} X_i$. Stein [6] shows that the probability that $S_1, S_2, \cdots, S_n$ are lying in the interval $(a, b)$ is of exponential order with $n$, where $a < 0 < b$ are arbitrary constants. In this case, we need not any conditions on $X_i$ except the trivial condition as $P(X_i = 0) < 1$.

On the other hand, if $X_i$ are bounded, from a result by Loève [3] it will be implied that $P(S_1 > a, \cdots, S_n > a)$ is of exponential order with $n$ provided $EX_i < 0$.

When the existence of variance of $X_i$ is assumed, the central limit theorem will be applicable. But, based on it, we can only see that the probability will converge to zero. Moreover, if a bounded condition in a sense is added, the order of the probability will be exponentially small with $n$ by a result due to Feller [1].

By the way, if $EX_i < 0$, using the results by Sparre Andersen [7] it is easily proved that

$$\sum_{n=1}^{\infty} P(S_1 > 0, \cdots, S_n > 0) < \infty,$$

then we can conclude that

$$P(S_1 > 0, \cdots, S_n > 0) = o\left(\frac{1}{n}\right).$$

This fact was noted by Prof. T. Kawata in his recent paper which discussed on queuing theory [2]. We shall investigate the problem when the boundedness of $X_i$ is not assumed. In §2 we shall give a counter example for the assertion that the order of the probability will be exponentially small with $n$, even if $EX_i < 0$. From the example, we can see the following fact: If we assume only $-\infty < EX_i < 0$, then we shall be able to assert only that the probability is $o(n^{-\varepsilon})$ as $n \rightarrow \infty$. Furthermore, in §3, we shall prove that the probability is smaller than $O(n^{-(1 + \sqrt{5})/2 + \varepsilon})$ under the condition that the mean and variance of $X_i$ exist, and the former is less than zero, where $\varepsilon$ is an arbitrary positive number.

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