ON CONFORMAL MAPPING OF A MULTIPLY-CONNECTED DOMAIN ONTO A CIRCULAR SLIT COVERING SURFACE

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§1. Introduction.

In the present paper we will concern ourselves with conformal mapping of a multiply-connected domain of finite connectivity onto a canonical covering surface whose boundary consists of whole circumferences and circular slits centred at the origin on the basic plane. We will discuss the existence of such a mapping function and its extremality. The purpose of our present investigation is an extension and an improvement of the results obtained in our previous papers [4] and [5].

§2. Preliminaries.

Let B be a multiply-connected domain of finite connectivity on the z-plane. We suppose that each component C_j $(j=1,\cdots,N)$ of its boundary C is a continuum. Let z_0, z_k^0 $(k=1,\cdots,N^0;\ N^0\geqq 0)$ and z_l^∞ $(l=1,\cdots,N^\infty;\ N^\infty\geqq 0)$ be arbitrarily preassigned $N^0+N^\infty+1$ points in B, and positive integers μ_k^0 and μ_l^∞ $(k=1,\cdots,N^0;\ l=1,\cdots,N^\infty)$ be given arbitrarily.[1] Let $\mathfrak{F}$ be the class of analytic functions $w=f(z)$ on B with the following properties:

(a) f has the only zeros z_k^0 $(k=1,\cdots,N^0)$ and the only poles z_l^∞ $(l=1,\cdots,N^\infty)$ with their orders μ_k^0 and μ_l^∞, respectively;[2]

(b) $$w=0,\ \infty\notin\overline{f(B)}-f(B);$$

(c) $$\left|\int_C \lg|f|\,d\arg f\right|<+\infty,$$

where the line integral means $\lim_{n\to\infty}\int_{\partial B_n}\lg|f|\,d\arg f$ with an exhaustion $\{B_n\}$ of B;

(d) $$f(z_0)=1.$$

Let B^* be a subdomain of B whose boundary C^* consists of components C_j^* $(j=1,\cdots,N)$, each being a simple analytic closed curve homotopic to C_j in $B-\sum_{k=1}^{N^0}\{z_k^0\}-\sum_{l=1}^{N^\infty}\{z_l^\infty\}$.[3] We define the *rotation number of the image of* C_j *about* $w=0$ *under* $f\in\mathfrak{F}$ by

$$\nu_j(f)=\frac{1}{2\pi}\int_{C_j^*}d\arg f \qquad (j=1,\cdots,N). \tag{1}$$

Then, it is easily verified by the argument principle that $\nu_j(f)$ $(j=1,\cdots,N)$

Received January 29, 1961.

1) Here the case $N^0=0$ or $N^\infty=0$ is permitted.

2) Of course, if $N^0=0$ or $N^\infty=0$, f has no zeros or no poles in B, respectively.

3) Here, in the case $N^0=0$ or $N^\infty=0$, the corresponding summations are taken to be vacuous, and the similar notes should be taken throughout the paper.