ON THE THEORY OF LINEAR DIFFERENCE-DIFFERENTIAL EQUATIONS

BY SHOHEI SUGIYAMA

Introduction.

Many authors have discussed the problems of linear difference-differential equations such that

$$\frac{dx(t)}{dt} = ax(t) + bx(t-1) + f(t) \tag{1}$$

for $0 \leqq t < \infty$. (Cf. [1]—[11], [15]—[22].)

However, few have been investigated concerning the problems of neutral systems. Hence, it may not be useless for us to consider a neutral linear equation such that

$$\frac{dx(t)}{dt} = ax(t) + bx(t-1) + c\frac{dx(t-1)}{dt} + f(t) \tag{2}$$

for $0 \leqq t < \infty$, where a, b, c are constant. Although neutral systems are essentially different from equations with $c=0$, the similar method is applicable for it.

In § 1, we are going to discuss the location of zeros of the characteristic equation corresponding to (2). In § 2, the existence of solutions of homogeneous equations will be studied. Non-homogeneous equations together with the fundamental solution will be discussed in § 3. In § 4, stability problem will be discussed as the application of the results in § 3. In § 5, perturbation method will be described for equations having a parameter.

The author would like to express his hearty thanks to Prof. Y. Komatu and Prof. T. Saito who looked over his paper and gave him a lot of advice during his investigation.

§ 1. Location of zeros of the characteristic equations.

In this section, we first consider the linear difference-differential equation

$$\frac{dx(t+1)}{dt} = ax(t+1) + bx(t) + c\frac{dx(t)}{dt} \tag{1.1}$$

with the constant coefficients a, b, and c, where we suppose that $c \neq 0$.

In the equation (1.1), if we put $x(t) = e^{st}$, we have

Received July 3, 1960.