ON THE EXISTENCE AND UNIQUENESS THEOREMS OF DIFFERENCE-DIFFERENTIAL EQUATIONS

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Introduction.

It is convenient to make use of differential equations in order to represent physical phenomena by certain functional relations. The differential equations, however, represent such phenomena by a relation between their present state and its instant change. Then, it is not apparent that the influence of past time does explicitly affect the present.

On the other hand, economic phenomena, generally speaking, have to be dependent on the past time, for example, on the influence before one year. In other words, we should better construct the equations by making use of the difference in order to forecast the result of the year before. They are, of course, to be observed in some physical phenomena.

For the sake of simplicity, we consider the following equation

$$F(t, x(t), x'(t), \cdots, x^{(m)}(t), x(t-1), x'(t-1), \cdots, x^{(n)}(t-1)) = 0,$$

where F, t, x represent scalars.

For $m > n$, it is supposed to be sovable for $x^{(m)}$, i.e.,

$$\frac{d^m x}{dt^m} = f(t, x(t), x'(t) \cdots, x^{(m-1)}(t), x(t-1), x'(t-1), \cdots, x^{(n)}(t-1)).$$

Putting $x(t) = y_1(t)$, $x'(t) = y_2(t)$, $\cdots$, $x^{(m-1)} = y_m(t)$, the above equation is reduced to

$$\frac{dy_1(t)}{dt} = y_2(t),$$
$$\frac{dy_2(t)}{dt} = y_3(t),$$
$$\vdots$$
$$\frac{dy_{m-1}(t)}{dt} = y_m(t),$$
$$\frac{dy_m(t)}{dt} = f(t, y_1(t), y_2(t), \cdots, y_m(t), y_1(t-1), y_2(t-1), \cdots, y_{n+1}(t-1)).$$

If we consider

$$y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{pmatrix}$$

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