A NOTE ON THE ENTROPY OF A CONTINUOUS DISTRIBUTION

By Hirohisa Hatori

1. The uniqueness of the expression $H=-\sum p_\nu \log p_\nu$ for the entropy of a discrete distribution has been discussed by Shannon [4] and Khintchin [2]. Goldman [1] has given an explanation of the entropy

$$H=-\int\cdots\int f(x_1,\cdots,x_n)\log f(x_1,\cdots,x_n)\,dx_1\cdots dx_n$$

of a continuous distribution on the basis of the discrete case. On the other hand Reich [3] has derived directly the expression for the information rate of a continuous distribution from some postulates. In this short paper we shall try to give another explanation of the entropy of a continuous distribution which is rather similar to the one in the discrete case.

2. Let $f(x_1,\cdots,x_n)$ denote the probability density function of the joint distribution of random variables $X_1,\cdots,X_n$. And we set

Postulate I. *The entropy $H(X_1,\cdots,X_n)$ of $(X_1,\cdots,X_n)$ is determined by f alone.*

Owing to this postulate we shall denote $H(X_1,\cdots,X_n)$ as $H(f)$.

Secondly, let $g_S(x_1,\cdots,x_n)$ denote the probability density function of the uniform distribution on a subset S with finite positive measure of the n-dimensional Euclidean space E_n.

Postulate II. *If f is the probability density function of an n-dimensional distribution where $f\not\equiv g_S(a.e.)$ and car. $(f)\subset S$, then $H(f)<H(g_S)$.*

Let $\phi(x_1,\cdots,x_k)$ be the probability density function of the radom variable $A\equiv(X_1,\cdots,X_k)$ and $\psi_{x_1,\dots,x_k}(x_{k+1},\cdots,x_n)$ be the conditional probability density function of the random variable $B\equiv(X_{k+1},\cdots,X_n)$ under the condition $X_1=x_1,\cdots,X_k=x_k$. We set

Postulate III. $\quad H(AB)=H(A)+H_A(B),$

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