AN EXTENSION OF KINTCHINE-OSTROWSKI'S THEOREM AND ITS APPLICATIONS

BY CHUJI TANAKA

1. Introdution. As an extension of Vitali's theorem, A. Kintchine and A. Ostrowski have proved

THEOREM ([1],[2],[3] p. 157). *Let* $\{f_n(z)\}$ $(n = 1, 2, \cdots)$ *be a sequence of functions regular and uniformly bounded in* $|z| < 1$. *If the sequence of boundary functions* $\{f_n(e^{i\theta})\}$ $(n = 1, 2, \cdots)$ *converges on a set* E *of* θ *of positive measure, then the sequence of* $\{f_n(z)\}$ *converges uniformly in the wider sense in* $|z| < 1$.

We shall first generalize this theorem as follows.

THEOREM 1. *Let* $\{f_n(z)\}$ $(n = 1, 2, \cdots)$ *be a sequence of functions regular in* $|z| < 1$ *and of uniformly bounded characteristic, i.e.*

$$\frac{1}{2\pi}\int_0^{2\pi}\log^+|f_n(re^{i\theta})|\,d\theta < M < +\infty \qquad (0 \leqq r < 1,\ n = 1, 2, \cdots), \tag{1.1}$$

where M *is a constant independent of* n. *If the sequence of boundary functions* $\{f_n(e^{i\theta})\}$ *converges on a set* E *of* θ *of positive measure, then the sequence* $\{f_n(z)\}$ *converges uniformly in the wider sense in* $|z| < 1$.

P. Montel has proved.

THEOREM ([4] p. 170). *Let* $f(s)$ $(s = \sigma + it)$ *be regular except at* $s = \infty$ *and bounded in the strip*: $\alpha \leqq \sigma \leqq \beta$. *If* $\lim_{t\to+\infty} f(\alpha + it) = a$, *then* $f(s)$ *tends uniformly to* a *as* $t \to +\infty$ *in the strip*: $\alpha \leqq \sigma \leqq \beta - \varepsilon$, ε *being any given positive constant.*

Before we establish an extension of Montel's theorem by theorem 1, we begin with

DEFINITION 1. *Let* $f(s)$ *be regular in the domain* D. *If*

$$\log^+|f(s)| \leqq h(s) \qquad \textit{for}\quad s \in D, \tag{1.2}$$

where $h(s)$ *is a harmonic function in* D, *then we say that* $f(s)$ *belongs to the*

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