THEOREMS ON SUBHARMONIC FUNCTIONS IN THE UNIT CIRCLE

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1. Let l_φ be a line through $e^{i\theta}$, making an angle φ $(-\pi/2 < \varphi < \pi/2)$ with the inner normal of $|z| = 1$ at $e^{i\theta}$. Then M. Tsuji [1] proved the following theorem.

Theorem. *Let*

$$w(z) = \int_{|a|<1} \log \left| \frac{1-\bar{a}z}{z-a} \right| d\mu(a),$$

where

$$\Omega(r) = \int_{|a|<r} d\mu(a) = O\Big(\frac{1}{(1-r)^{\lambda}}\Big), \qquad 0 < \lambda < 1.$$

Then there exists a set E of measure 2π on $|z| = 1$, such that if $e^{i\theta} \in E$, then for almost all ψ,

$$\lim_{z \to e^{i\theta}} w(z) = 0,$$

when $z \to e^{i\theta}$ along $l_\psi(e^{i\theta})$.

Let $u(z)$ be a subharmonic function in $|z| < 1$ such that

$$\int_0^{2\pi} |u(re^{i\theta})| \, d\theta = O(1), \qquad 0 \leqq r < 1,$$

and put

$$L(u, r) = \frac{1}{2\pi} \int_0^{2\pi} u(re^{i\theta}) \, d\theta,$$

then $L(u, r)$ is an increasing convex function of $\log r$, and Tsuji proved the following theorem.

Theorem. *Let $u(z)$ be a subharmonic function in $|z| < 1$, such that*

$$\int_0^{2\pi} |u(re^{i\theta})| \, d\theta = O(1), \qquad \frac{d}{dr} L(u, r) = O\Big(\frac{1}{(1-r)^{\lambda}}\Big), \qquad 0 < \lambda < 1.$$

Then there exists a set E of measure 2π on $|z| = 1$, such that if $e^{i\theta} \in E$, then for almost all ψ,

$$\lim_{z \to e^{i\theta}} u(z) = u(e^{i\theta}) \neq \infty$$

exists, when $z \to e^{i\theta}$ along $l_\psi(e^{i\theta})$.

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