ON THE SYSTEM OF NON-LINEAR DIFFERENTIAL EQUATIONS WITH PERIODIC COEFFICIENTS

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§ **1.** Recently the author has investigated the behaviour of the solution of the non-linear differential equation

$$\frac{dy}{dx} = \sum_{k=1}^{\infty} f_k(x)\, y^k$$

where $f_k(x)$ are uniform and holomorphic in the domain $0 < |x| < r$, and obtained an analytical expression of the solution valid around $x = 0$.[1)]

The method of proof used there can easily be generalized for the system of non-linear differential equations

$$\text{(A)} \qquad \frac{dy_j}{dx} = \sum_{k_1+\cdots+k_n \geqq 1} f_{j,k_1\cdots k_n}(x)\, y_1^{k_1} \cdots y_n^{k_n}, \qquad j = 1, \cdots, n,$$

with $f_{j,k_1\cdots k_n}(x)$ uniform and holomorphic in $0 < |x| < r$, or, what is the same thing, for the system

$$\text{(B)} \qquad \frac{dx_j}{dt} = \sum_{k_1+\cdots+k_n \geqq 1} a_{j,k_1\cdots k_n}(t)\, x_1^{k_1} \cdots x_n^{k_n}, \qquad j = 1, \cdots, n,$$

with $a_{j,k_1\cdots k_n}(t)$ periodic in t.

In the present paper, we consider the system (B), and establish the analytical expression of its solutions.

§ **2.** Let the system of differential equations

$$(1) \qquad \frac{dx_j}{dt} = \sum_{k=1}^{n} a_{j,k}(t)\, x_k + \sum_{k_1+\cdots+k_n \geqq 2} a_{j,k_1\cdots k_n}(t)\, x_1^{k_1} \cdots x_n^{k_n}, \qquad j = 1, \cdots, n,$$

be given, where $k_1, \cdots, k_n$ are non-negative integers, $a_{j,k}(t)$ and $a_{j,k_1\cdots k_n}(t)$ are periodic functions of t with period 1 holomorphic for $-\infty < t < \infty$, and the power series in the right-hand members are convergent for

$$-\infty < t < \infty, \quad |x_j| < \rho, \quad \rho > 0, \qquad j = 1, \cdots n.$$

Without loss of generality, we may suppose that the matrix $\| a_{j,k}(t) \|$ is of the following form:

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