FURTHER SUPPLEMENT TO "ON TRANSFERENCE OF BOUNDARY VALUE PROBLEMS"

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In a previous paper [1] it has been shown that for some simple domains two main kinds of boundary value problems in potential theory are transferable each other by means of elementary operations. And then in another paper [2] it has been supplemented that the relations for transference can be readily verified also by deriving separately the explicit formulas for both problems.

However, in these papers explicit use has been made of well-known but special formulas for solving two kinds of boundary value problems with respect to a circle. In the present Note, considering the same topics once again, we shall derive the results without any reference to these special formulas. The present method may probably suggest a way to obtain analogous results for more general types of basic domains.

1. Rectilinear slit domain.

Theorem A. *Let D be a basic domain laid on the $z=x+iy$-plane whose boundary C is a segment defined by*

$$C: \qquad x=0, \qquad -1 \leqq y \leqq +1.$$

Let $u(z)=\Re f(z)$ and $v(z)=\Re g(z)$, $f(z)$ and $g(z)$ being analytic, be the solutions of Dirichlet and Neumann problems, respectively, with related boundary conditions

$$u=\pm V^{\pm}(y), \quad \frac{\partial v}{\partial \nu}=V^{\pm}(y) \qquad \text{for} \quad z=\pm 0+iy \quad (-1<y<+1),$$

$\partial/\partial\nu$ denoting the differentiation along inward normal; the condition for solvability of the latter problem, i.e.

$$\int_{-1}^{1}(V^{+}(y)+V^{-}(y))\,dy=0,$$

is, of course, supposed to be valid. Then there holds a connecting relation

$$f(z)=g'(z)+i\rho+\frac{\sigma z+i\tau}{\sqrt{1+z^2}}$$

or

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