# SOME CLASSES OF POSITIVE SOLUTIONS OF $\Delta u = Pu$ ON RIEMANN SURFACES, I

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## § 0. Introduction.

In our earlier paper [1] we had introduced a method to classify the Riemann surfaces into the various types making use of the following leading idea: Under what conditions does there exist a solution of a partial differential equation of elliptic type $\Delta u = Pu$ on a given Riemann surface?

Recently Lauri Myrberg [1][2] showed that on every Riemann surface there always exists the Green function of $\Delta u = Pu$. This is an elegant result in this tendency.

Martin topology played an important role in Heins' investigation for the structure of ideal boundary and for a class of positive harmonic functions on an end in Heins' sense. An analogue of Heins' investigation was stated and a maximality of a class of positive harmonic functions on a subregion with non-compact relative boundary was established in our papers [2][3].

In the present paper we shall investigate the structure of the ideal boundary using the positive solutions of $\Delta u = Pu$ instead of positive harmonic functions. In the way of construction of the theory we are obliged to add a more restriction than in the harmonic case in order to establish a full parallelism between the theory in Heins' paper and that in the present paper. In harmonic case, when F belongs to the class $O_G$, the maximum-minimum principle in the extended sense for any bounded harmonic function holds and we can conclude that each minimal positive harmonic function in Martin's sense on any Heins' end is obtained by a suitable limiting process $m \to \infty$ ($p_m \to$ ideal boundary) for the Green function $G(z, p_m)$ on a given end.

Riemann surfaces F in considerations are the ones in the sense of Weyl-Radó. Differential equation considered here is the following type

$$(A) \quad \frac{\partial^2}{\partial x^2} u(x,y) + \frac{\partial^2}{\partial y^2} u(x,y) = P(x,y)\, u(x,y),$$

where $z = x+iy$ is a local parameter at a point $p$ on F, $P(x,y)$ is a real continuous function of $(x,y)$ being except at most at a countable set of zero-points with accumulation points lying only on the ideal boundary and having continuous partial derivatives. We assume moreover that if we change the local parameter $z$ to $z'$, then $P(z)$ changes as follows:

$$P(z) = P(z') \left| \frac{dz'}{dz} \right|^2.$$

For this type of differential equation (A) we can prove the existence of the Green function and the solvability and uniqueness of the first boundary value problem on compact subsurface. Moreover Harnack's convergence theorem is valid. For the precise formulations one can refer to the well-known classical papers. Recently L. Myrberg [1][2] gave a precise method of formulations.

## § 1. Generalities and known results.

Let $\{F_n\}$ $n = 0,1,2,\cdots$ be an exhaustion of F in the ordinary sense. Let $F_n$ have a compact analytic curve $\Gamma_n$ as its relative boundary.

Let $G_n(z, \zeta)$ be the Green function of (A) on $F_n$ satisfying the following conditions:

1. $\Delta G_n(z,\zeta) = P(z) \cdot G_n(z,\zeta)$ on $F_n$,
2. $G_n(z,\zeta)$ has continuous partial derivatives of second order on $F_n - \zeta$,