# A SUPPLEMENT TO "ON TRANSFERENCE OF BOUNDARY VALUE PROBLEMS"

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In a recent paper[1)] it has been shown that for some simple domains Dirichlet and Neumann problems are readily transferable each other by means of elementary operations. Since attention has been restricted to give the connection between the solutions of both boundary value problems, the explicit formulas for the solutions have not been brought forwards in practical forms. However, it is possible to derive them separately also in elementary ways, what will be supplemented in the present paper.

**1.** Rectilinear slit domain.

Let the basic domain be the whole $z$-plane slit along a rectilinear segment

$$\mathfrak{R}z=0, \quad -1 \leqq \mathfrak{J}z \leqq +1,$$

and let first the boundary condition of a Neumann problem be assigned in the form

$$\frac{\partial v}{\partial \nu}(\pm 0+iy)=V^{\pm}(y) \qquad (-1<y<1),$$

$$\int_{-1}^{1}(V^{+}(y)+V^{-}(y))\,dy=0.$$

It is solved, as stated in the previous paper, by $v(z)=\mathfrak{R}g(z)$, where $g(z)$ is defined by

$$g(z) = c-\frac{1}{\pi}\left\{\int_{-\pi/2}^{\pi/2}\mathcal{V}^{-}(\varphi)-\int_{\pi/2}^{3\pi/2}\mathcal{V}^{+}(\varphi)\right\}\cos\varphi \lg\frac{1}{e^{i\varphi}-\mathfrak{z}}\,d\varphi,$$

$$\mathcal{V}^{\pm}(\varphi)=V^{\pm}(y)\equiv V^{\pm}(\sin\varphi),$$

$$\mathfrak{z}=z-\sqrt{1+z^2},$$

$c$ being any constant and the square root representing such a branch that $z=\infty$ corresponds to $\mathfrak{z}=0$.

Now substitute an integration variable $\eta$ defined by

$$\eta=\sin\varphi, \quad e^{i\varphi}=i\eta\pm\sqrt{1-\eta^2},$$

where the upper and lower of the double sign is taken for $-\pi/2<\varphi<\pi/2$ and for $\pi/2<\varphi<3\pi/2$, respectively, and $\sqrt{1-\eta^2}$ is supposed to represent always a non-negative real number. In view of the relation

$$\frac{1}{e^{i\varphi}-\mathfrak{z}} = -\frac{1}{2}\frac{1}{z-i\eta}\left(1+(z+\sqrt{1+z^2})(i\eta\pm\sqrt{1-\eta^2})\right),$$

a required formula for the solution of the Neumann problem is obtained in the form

$$g(z)=c-\frac{1}{\pi}\int_{-1}^{1}\Big\{(V^{+}(\eta)+V^{-}(\eta))\lg\frac{1}{z-i\eta}$$
$$+V^{+}(\eta)\lg(1+(z+\sqrt{1+z^2})(i\eta+\sqrt{1-\eta^2}))$$
$$+V^{-}(\eta)\lg(1+(z+\sqrt{1+z^2})(i\eta-\sqrt{1-\eta^2}))\Big\}\,d\eta.$$

Let next the boundary condition of a Dirichlet problem be assigned in the form

$$u(\pm 0+iy)=U^{\pm}(y) \qquad (-1<y<1).$$

It is solved by $u(z)=\mathfrak{R}f(z)$, where $f(z)$ is defined by

$$f(z) = \frac{1}{2\pi}\left\{\int_{-\pi/2}^{\pi/2}\mathcal{U}^{-}(\varphi)+\int_{\pi/2}^{3\pi/2}\mathcal{U}^{+}(\varphi)\right\}\frac{e^{i\varphi}+\mathfrak{z}}{e^{i\varphi}-\mathfrak{z}}\,d\varphi,$$

$$\mathcal{U}^{\pm}(\varphi)=U^{\pm}(\eta)\equiv U^{\pm}(\sin\varphi),$$

$$\mathfrak{z}=z-\sqrt{1+z^2},$$