A CHARACTERIZATION OF HOMOTOPICALLY LABIL POINTS

By Hiroshi NOGUCHI

(Comm. by H. Tôyama)

1. The concept of the labil point and the lability of a space was defined by H. Hopf and E. Pannwitz [2]. In a recent work, K. Borsuk and J. W. Jaworowski introduced an analogous concept which is called a homotopically labil point [1,pp.159-160].

The object of the present paper is to give a characterization of the homotopically labil point (in section 3). In section 2 we shall give some remarks concerning the definitions of the labil point and the homotopically labil point. In section 4 we shall show a geometrical behaviour of the homotopically labil points in 2-dimensional homogeneous complexes.

2. Let M be a space and I be the interval $0 \leqq t \leqq 1$. A mapping $f(x,t)$ which is defined in the Cartesian product $M \times I$ will be called a deformation of M whenever it satisfies the following conditions;

$$f(x,t) \in M \text{ for every } (x,t) \in M \times I$$

$$f(x,0) = x \text{ for every } x \in M.$$

The concept of the labil point owing to H. Hopf and E. Pannwitz [2, p.434] can be formulated as follows;

(2.1) DEFINITION. A point a of a space M is labil whenever for every neighbourhood U of a there exists a deformation $f(x,t)$ of M satisfying the following conditions;

(1) $f(x,t) = x$ for every $(x,t) \in (M-U) \times I$

(2) $f(x,t) \in U$ for every $(x,t) \in U \times I$

(3) $f(U,1) \neq U$.

(2.2) REMARK. We see easily that the property of being a labil point is a local one.

The concept of the lability of a space owing to H. Hopf and E. Pannwitz [2,p.434] can be formulated as follows;

(2.3) DEFINITION. A metric space M is labil if for every $\varepsilon > 0$ there exists a deformation $g(x,t)$ of M satisfying the following conditions;

(4) $\rho(x, g(x,t)) < \varepsilon$ for every $(x,t) \in M \times I$,

(5) $g(M,1) \neq M$,

where ρ denotes the metric of M.

(2.4) REMARK. It is remarked by H. Hopf and E. Pannwitz [2,p.434] that for compactum M, M is labil if, and only if, M has at least one labil point. In this statement the assumption that M is compact cannot be removed. In fact, consider in the euclidean plane the set $M = S - (1,0)$, where S denotes the unit circle having $(0, 0)$ as center. Then M is labil, but has not any labil point.

On the other hand, this example also shows that for non-compact space the lability is not a local property. In fact, M is locally homeomorphic to S, but S is evidently not labil.

The concept of the homotopically labil point owing to K. Borsuk and J. W. Jaworowski [1] can be formulated as follows;

(2.5) DEFINITION. A point a of a space M is homotopically labil whenever for every neighbourhood U of a there exists a deformation $f(x,t)$ of M satisfying the following conditions;

(6) $f(x,t) = x$ for every $(x,t) \in (M-U) \times I$,

(7) $f(x,t) \in U$ for every $(x,t) \in U \times I$

(8) $f(x,1) \neq a$ for every $x \in M$.