M. OZAWA
KŌDAI MATH. SEM. REP.
29 (1978), 308–321

ON THE EXISTENCE OF PRIME PERIODIC ENTIRE FUNCTIONS

BY MITSURU OZAWA

1. In our earlier paper [7] we proved the existence of a periodic entire function which is prime. This gives an affirmative answer to an open problem given by Gross [5]. Recently Baker and Yang [2] have discussed the same problem, having emphasized the growth of the constructed function. Their example is really not only of infinite order but also of infinite hyperorder. In the same point of view there still remains a problem whether there is a prime periodic entire function of given growth. Here of course the given order should not be less than one. Our theorems give a partial answer to the above problem.

As already remarked in [7] we must seek for a prime periodic entire function among the class of entire functions $h(e^z)$, where $h(w)$ is a one-valued regular function in $0<|w|<\infty$, having essential singularities at $w=0$ and $w=\infty$. Of course this is not sufficient for the problem as remarked there.

THEOREM 1. *There is an entire periodic function of order* $\rho\,(1\leqq\rho<\infty)$, *which is prime.*

THEOREM 2. *There is an entire periodic function of hyperorder* $\rho\,(1\leqq\rho<\infty)$, *which is prime.*

Here the order and the hyperorder of f mean

$$\varlimsup_{r\to\infty}\frac{\log m(r, f)}{\log r} \quad \text{and} \quad \varlimsup_{r\to\infty}\frac{\log\,\log m(r, f)}{\log r}$$

respectively.

2. *Proof of Theorem* 1. We firstly construct an entire function $\Pi_1(w)$ so that

$$\log M(r, \Pi_1)\sim(\log r)^{\rho}, \quad 1<\rho<\infty^{-}.$$

In this case we may assume that the absolute moduli of zeros of $\Pi_1(w)$ are greater than 1. There are infinitely many such functions. Let $\Pi_2(w)$ be the same as in [7]. Let $F(z)$ be

Received April 13, 1977.