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A CERTAIN DERIVATIVE IN FIBRED RIEMANNIAN SPACES, AND ITS APPLICATIONS TO VECTOR FIELDS

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Introduction. Recently, Ishihara [1] studied vector fields in fibred Riemannian spaces with 1-dimensional fibre. The main purpose of the present paper is to study these problems in fibred Riemannian spaces with higher dimensional fibre.

For this purpose, we define a kind of derivatives which are closely related to Lie derivative, to describe some properties of vector fields in fibred Riemannian spaces with higher dimensional fibre.

In the first section, we shall give some preliminaries for fibred Riemannian spaces following to the sense of Ishihara-Konishi [2]. In the second section, we shall derive the so-called structure equations of fibred Riemannian spaces, which were mainly obtained in a previous paper [9]. In the third section, we shall define the (*)-Lie derivative for later use. Section 4, 5 and 6 are devoted to the study of vector fields, Killing, affne Killing and projective Killing respectively.

§ 1. Preliminaries on fibred spaces

In this section, we shall recall definitions and properties concerning fibred spaces in the sense of Ishihara-Konishi [2].

Let $\tilde{M}$ and M be two differentiable manifolds of dimension r and n respectively, where $s=r-n>0$, and suppose that there exists a differentiable mapping $\pi : \tilde{M} \to M$ which is onto and maximal rank n everywhere. Throughout the paper, the differentiability of manifolds, mappings and geometric objects we discuss are assumed to be of C^∞. The manifolds we discuss are assumed to be connected. Then the inverse image $\pi^{-1}(P)$ of any point P of M is an s-dimensional submanifold of $\tilde{M}$, which is called the *fibre* over P and denoted by F_P, or simply by F. Moreover we assume that each fibre is connected. Such a set $\{\tilde{M}, M, \pi\}$ is called a *fibred space*, $\tilde{M}$ the *total space*, M the *base space* and π the *projection*.

Let there be given a Riemannian metric $\tilde{g}$ in $\tilde{M}$ of a fibred space $\{\tilde{M}, M, \pi\}$. Then the set $\{\tilde{M}, M, \tilde{g}, \pi\}$ is called a *fibred space with Riemannian metric* $\tilde{g}$ and

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