ON THE LINEAR TRANSLATABLE STOCHASTIC FUNCTIONAL EQUATION.

By Takuji ONOYAMA

1. Introduction.

Our problem is how to study a special solution of the linear translatable stochastic functional equation;

$$(1.1) \qquad \Lambda f(x,\omega) \equiv \int_0^1 f(x+t,\omega)\,d\varphi(t) = g(x,\omega),$$

where

1°, Λ is a linear translatable operator,

2°, $g(x,\omega)$ is a given strictly stationary stochastic process, and

3°, $\int_0^1 \cdot\, d\varphi$ is defined as Bochner's integral according to the operator Λ.

The object of this paper is to study especially the case when zero points of generating function $G(\lambda)$ ($\Lambda e^{\lambda x} \equiv G(\lambda) e^{\lambda x}$) of Λ are only pure imaginary, because other cases are not so difficult.

Here we have to note that N.Wiener's[1] and T.Kitagawa's[2] method in the pure functional scheme are not always adoptable as they are.

2. Preliminary.

1°, We put here:

$$(2.1) \qquad G(\lambda) = \int_0^1 e^{\lambda t}\,d\varphi(t)$$

where

$G(\lambda)$ is an integral function, and let λ_0 be a zero point of order k ($k \geqq 0$) of $G(\lambda)$, then we can write following:

$$(2.2) \qquad \frac{(\lambda-\lambda_0)^k e^{(\lambda-\lambda_0)h}}{G(\lambda)} = \sum_{s=0}^{\infty} B^k_{s,\lambda_0}(h)(\lambda-\lambda_0)^s$$

$$(|\lambda-\lambda_0| < \rho(\lambda_0))$$

with $\rho(\lambda_0)$ which is the distance from λ_0 to the other nearest zero point of $G(\lambda)$ on the imaginary axis.

$\{B^k_{s,\lambda_0}(h)\}$ is the sequence of the generalized Bernoulli's polynomials[3].

Lemma 1[4]

$$(2.3) \qquad \Lambda B^k_{s,\lambda_0}(h)e^{\lambda_0 h} = \begin{cases} \dfrac{h^{s-k}}{(s-k)!}e^{\lambda_0 h}, & (s=k,k+1,\cdots) \\ 0 & (s=0,1,\cdots k-1) \end{cases}$$

$$(2.4) \qquad B^k_{s,\lambda_0}(h_1+h_2) = \sum_{\nu=0}^{s} \frac{h_2^{\nu}}{\nu!} B^k_{s-\nu,\lambda_0}(h_1) \qquad (s=0,1,2,\cdots)$$

2°, Regarding $f(x,\omega)$, $g(x,\omega)$ as two strictly stationary stochastic processes, we have the following definition:

$$(2.5) \qquad \text{distance } (f,g) \equiv \|f-g\| \equiv \sqrt{\int_\Omega |f(x,\omega)-g(x,\omega)|^2\,dP}$$

We call this ω norm.

Lemma 2.[5] A strictly stationary stochastic process $g(x,\omega)$ and its autocorrelation coefficient $R_{|s-t|}$ are represented as follows.

$$(2.6) \qquad g(x,\omega) = \int_{-\infty}^{\infty} e^{i\lambda x}\,dS(\lambda,\omega)$$

$$(2.7) \qquad R_{|s-t|} = \int_{-\infty}^{\infty} e^{i\lambda(s-t)}\,dF(\lambda)$$

$$-1 \leqq R_{|s-t|} \leqq 1,$$

where $S(\lambda,\omega)$ is a differential process, and $F(\lambda)$ is a spectre function defined by $S(\lambda,\omega)$.

Lemma 3[6] If $\int_{-\infty}^{\infty} |\varphi(t)|^2\,dF(\lambda) < \infty$, then

$$\int_{-\infty}^{\infty} \varphi(\lambda)e^{it\lambda}\,dS(\lambda,\omega) \quad \text{and} \quad R_{|t|} = \int_{-\infty}^{\infty} e^{i\lambda t}\,dF(\lambda)$$

exist.

Let λ_i ($i=0,1,2,\cdots$) be zero points of $G(\lambda)$ on imaginary axis, and be non dense in any interval on imaginary axis, then the interval $(-\infty,\infty)$ can be devided into the direct sum $(I_1 \oplus I_2 \oplus \cdots)$ of enumerable subintervals I_i $(i=0,1,2,\cdots)$ by $\rho(\lambda_i)$ $(i=0,1,2,\cdots)$ in (2.2).