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ON A DUALITY THEOREM OF ABELIAN VARIETIES OVER HIGHER DIMENSIONAL LOCAL FIELDS

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Abstract

In this paper, we prove a duality theorem of abelian varieties over higher dimensional local fields under some conditions. It might be a one of generalization of the classical Tate duality theorem of abelian varieties over local fields.

1. Introduction

In this paper we show a duality theorem of the Galois cohomology groups related to abelian varieties over higher dimensional local fields. It might be a good generalization of the classical Tate duality of abelian varieties over usual local fields.

In order to understand our situation, we recall the classical Tate duality. Let K be a local field of characteristic 0, that is, a complete discrete valuation field with finite residue field. Let A be an abelian variety over K, and A^t the dual variety of A over K. The Weil-Barsotti formula makes the following identification (cf. [7, Théorème 6, §16, No. 17 Chapitre VII])

$$A^t(K) = \mathrm{Ext}_K^1(A, \boldsymbol{G}_m).$$

Then we know that the canonical pairing

$$H^r(K, A) \times H^{1-r}(K, A^t) \to \boldsymbol{Q}/\boldsymbol{Z} \quad (r = 0, 1)$$

induces isomorphisms

$$H^1(K, A) = A^t(K)^*,$$
$$H^1(K, A^t) = A(K)^*,$$

where $A^t(K)^*$ and $A(K)^*$ stand for the Pontrjagin duals of them. The proof of the fact above deeply depends on the local class field theory.

The above remark and Kato's higher dimensional local class field theory might lead us to the idea to establish the higher Tate duality. Actually one can prove a duality theorem between the torsion parts of abelian varieties over

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