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A NOTE ON NECESSARY CONDITIONS OF HYPOELLIPTICITY FOR SOME CLASSES OF DIFFERENTIAL OPERATORS WITH DOUBLE CHARACTERISTICS

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Abstract

We construct explicit formulas for fundamental solutions and global non-smooth solutions at degenerate points of some classes of differential operators with double characteristics. A new elementary proof for non-hypoellipticity is given.

§1. Introduction

In this paper we will construct explicit formulas for fundamental solutions and global non-smooth solutions at degenerate points of the following operator

$$G_{k,c}^{a,b} = X_2 X_1 + icx^{k-1}\frac{\partial}{\partial y},$$

where $(x, y) \in \boldsymbol{R}^2$; $a, b, c \in \boldsymbol{C}$, $\operatorname{Re} a \cdot \operatorname{Re} b \neq 0$; $i = \sqrt{-1}$; k is a positive integer, and $X_1 = (\partial/\partial x) - ibx^k(\partial/\partial y)$, $X_2 = (\partial/\partial x) - iax^k(\partial/\partial y)$. The operator $G_{k,c}^{a,b}$ was studied in [1] when k is odd and in [2] when k is even. For more complete references and generalization we refer to [3], [4], [5], [6], [7] and therein references. We will treat only the case $\operatorname{Re} a < 0$. The case $\operatorname{Re} a > 0$ can be considered analogously. Recently in [8], [9] we considered a model of the Grushin operator, that is the case when $a = -1$, $b = 1$, and the Kohn-Laplacian on the Heisenberg group. The paper is organized as follows. In §2 we give some definitions of notations used in the paper, and establish some auxiliary lemmas. In §3 we state and prove the main results.

§2. Auxiliary lemmas

We will use the following notation

$$(z, m) = z(z+1)\cdots(z+m-1) = \frac{\Gamma(z+m)}{\Gamma(z)} \quad \text{for } z \in \boldsymbol{C}, m \in \boldsymbol{N}.$$

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