

## SYMMETRIC WEIGHTS AND S-REPRESENTATIONS

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### Abstract

We study irreducible representations of compact Lie groups relating an algebraic condition (the highest weight  $\lambda$  is “symmetric”, i.e., in any simple factor all non zero  $\langle \lambda, \alpha \rangle$  are equal, for any positive root  $\alpha$  and any invariant inner product) with a geometric one (for all orbits, the  $d$ -th osculating space coincides with the representation space).

We prove that, if  $d = 2$  and  $\lambda$  is symmetric, the irreducible representation with highest weight  $\lambda$  corresponds to the isotropy representation of a symmetric space.

### 1. Introduction

Let  $K$  be a compact Lie group and  $\phi$  a faithful irreducible orthogonal representation. Our aim is to investigate the interplay between algebraic properties of the weight system of  $\phi$  and geometric properties of the representation  $\phi$ .

Among orthogonal representations, a crucial rôle in submanifold geometry is played by the isotropy representations of symmetric spaces, called *s-representations*. Indeed the principal orbits of s-representations are isoparametric and the singular ones are their focal manifolds. Moreover all orbits of s-representations are *taut* [2].

The s-representations are strictly related to another class of orthogonal representations whose definition is geometrically more appealing: the *polar representations*. A representation of a compact Lie group  $K$  on vector space  $V$  is polar if there is a linear subspace  $\Sigma \subset V$  that meets all orbits of  $K$  and every time it meets an orbit of  $K$ , it meets it perpendicularly. It is not difficult to see that any s-representation is polar. Moreover it is still true that any orbit of a polar representation is taut, as it follows from results of Conlon [4] together with ones of Bott and Samelson [2].

On the other hand, Dadok [6] classified all irreducible polar representations and observed that some of them are s-representations and that, those that are not, have the same orbits as s-representations. For his classification, Dadok associated to any irreducible representation with highest weight  $\lambda$ , an integer  $k(\lambda)$ .

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