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ON SPECIAL VALUES OF STANDARD L-FUNCTIONS ATTACHED TO VECTOR VALUED SIEGEL MODULAR FORMS

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1. Introduction

Let V be a vector space of dimension $n \in \mathbf{Z}_{>0}$ over $\mathbf{C}$ and $\mathrm{sym}^l(V)$ the l-th symmetric tensor product of V with $l \in \mathbf{Z}_{\geq 0}$. For $k \in \mathbf{Z}_{\geq 0}$, let f be a $\mathrm{sym}^l(V)$-valued Siegel modular form of type $\det^k \otimes \mathrm{sym}^l$ with respect to $Sp(n, \mathbf{Z})$ (size $2n$). Suppose f is a cuspform and an eigenform (i.e., a non-zero common eigenfunction of the Hecke algebra). Then we define the standard L-function attached to f by

$$(1.1) \qquad L(s, f, \underline{\mathrm{St}}) := \prod_p \left\{ (1 - p^{-s}) \prod_{j=1}^{n} (1 - \alpha_j(p) p^{-s})(1 - \alpha_j(p)^{-1} p^{-s}) \right\}^{-1},$$

where p runs over all prime numbers and $\alpha_j(p) (j = 1, \dots, n)$ are the Satake p-parameters of f. The right-hand side of (1.1) converges absolutely and locally uniformly for $\mathrm{Re}(s) > n + 1$. We put

$$\Lambda(s, f, \underline{\mathrm{St}}) := \Gamma_{\mathbf{R}}(s + \varepsilon) \Gamma_{\mathbf{C}}(s + k + l - 1) \prod_{j=2}^{n} \Gamma_{\mathbf{C}}(s + k - j) L(s, f, \underline{\mathrm{St}})$$

with

$$\Gamma_{\mathbf{R}}(s) := \pi^{-s/2} \Gamma\left(\frac{s}{2}\right), \quad \Gamma_{\mathbf{C}}(s) := 2(2\pi)^{-s} \Gamma(s),$$

and

$$\varepsilon := \begin{cases} 0 & \text{for } n \text{ even,} \\ 1 & \text{for } n \text{ odd.} \end{cases}$$

Then by Takayanagi [9, Theorem 2, Theorem 3], we have:

If $k, l \in 2\mathbf{Z}$, $k > 0$, $l \geq 0$, then $\Lambda(s, f, \underline{\mathrm{St}})$ has a meromorphic continuation to the whole s-plane and satisfies the functional equation

$$\Lambda(s, f, \underline{\mathrm{St}}) = \Lambda(1 - s, f, \underline{\mathrm{St}}),$$

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