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ON RECURRENCE FOR SELF-SIMILAR ADDITIVE PROCESSES

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1. Introduction

In our paper a stochastic process is called an additive process if it is a stochastically continuous process with independent increments and has right-continuous sample functions with left limits a.s. Self-similar additive processes constitute an important class of additive processes which are not assumed to be time-homogeneous. But they have not been studied except in some papers, e.g. [2], [3], [4], and [5]. We investigated their transience and recurrence in [3] and [5]. The dichotomy of recurrence and transience for this class of processes is known (see [3]). But a criterion for recurrence and transience has not been found. As an important example, there is a strictly stable process on $\boldsymbol{R}^d$, which is a self-similar Lévy process, that is, a self-similar time-homogeneous additive process. It is recurrent if its index α satisfies $\max\{1,d\} \le \alpha \le 2$, where its exponent is α^{-1} if we regard it as a self-similar additive process (see the definition below). So we attempted to find a new method to prove recurrence for strictly stable processes with index $\max\{1,d\} \le \alpha \le 2$ without using time-homogeneity. We succeeded in our attempt and we could find recurrence conditions showing great differences between self-similar additive processes and Lévy processes. We note that this problem cannot be solved by using the existing methods because of the difficulty caused by the fact that the expected occupation times on open sets containing 0 cannot determine recurrence (see [3]).

A self-similar additive process is defined by the following.

DEFINITION. A stochastic process $\{X_t : t \ge 0\}$ on $\boldsymbol{R}^d$, which is defined on a probability space $(\Omega, \mathscr{F}, P)$, is called a self-similar additive process, or a process of class L, with exponent $H > 0$ if it satisfies the following conditions:

(i) $\{X_{ct}\}$ and $\{c^H X_t\}$ have the same finite-dimensional distributions for every $c > 0$,

(ii) $X_{t_1} - X_{t_0}, X_{t_2} - X_{t_1}, \dots, X_{t_n} - X_{t_{n-1}}$ are independent for any n and any choice of $0 \le t_0 < t_1 < t_2 < \cdots < t_n$,

(iii) almost surely X_t is right-continuous in $t \ge 0$ and has left limits in $t > 0$.

Throughout this paper let $\{X_t\}$ be a self-similar additive process on $\boldsymbol{R}^d$ with

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