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# HYPERBOLIC HYPERSURFACES IN THE COMPLEX PROJECTIVE SPACES OF LOW DIMENSIONS

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## §1. Introduction

There have been a number of results for hyperbolic hypersurfaces in the complex projective spaces (cf. [AS], [BG], [D], [K], [MN], [N], [S] and [Z]). In particular, J. P. Demailly [D] constructed a remarkable example of hyperbolic hypersurfaces of degree 11 in $\boldsymbol{P}^3(\boldsymbol{C})$. On the other hand, the author [S] gave hyperbolic hypersurfaces of degree $13^n$ in $\boldsymbol{P}^n(\boldsymbol{C})$ whose complements are complete hyperbolic and hyperbolically imbedded in $\boldsymbol{P}^n(\boldsymbol{C})$. In this paper, we give hyperbolic hypersurfaces in the complex projective spaces of dimension 2, 3 and 4. For example, we construct hyperbolic hypersurfaces in $\boldsymbol{P}^3(\boldsymbol{C})$ of degree 31 whose complements are complete hyprebolic and hyperbolically imbedded in $\boldsymbol{P}^3(\boldsymbol{C})$, and hyperbolic hypersurface of degree 36 in $\boldsymbol{P}^4(\boldsymbol{C})$.

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## §2. A holomorphic mapping into a hypersurface in $\boldsymbol{P}^n(\boldsymbol{C})$

Let $n$, $q$ and $d$ be positive integers such that $q \geq n+1$ and $d \geq (q-1)^2$. Let $V$ be a set of $q$ column vectors in $\boldsymbol{C}^{n+1}$. We make the following assumptions.

(A1) The vectors in $V$ are in general position.

(A2) Take any $k$ with $0 \leq k \leq \min\{n, q-n-2\}$. Then, for any distinct vectors $\boldsymbol{v}_0, \ldots, \boldsymbol{v}_n, \boldsymbol{u}_0, \ldots, \boldsymbol{u}_k$ in $V$ and any $d$-th roots of $\omega_0, \ldots, \omega_k$ of $-1$, the $n+1$ vectors $\boldsymbol{v}_j - \omega_j \boldsymbol{u}_j$ $(0 \leq j \leq k)$ and $\boldsymbol{v}_j$ $(k+1 \leq j \leq n)$ are linearly independent.

(A3) Take any $k$ with $1 \leq k \leq \min\{n, q-n-1\}$. Then, for any distinct vectors $\boldsymbol{v}_0, \ldots, \boldsymbol{v}_n, \boldsymbol{u}_1, \ldots, \boldsymbol{u}_k$ in $V$

$$\sum_{j=1}^{k} \left\{ \frac{\det(\boldsymbol{u}_j, \boldsymbol{v}_1, \ldots, \boldsymbol{v}_n)}{\det(\boldsymbol{v}_0, \boldsymbol{v}_1, \ldots, \boldsymbol{v}_n)} \right\}^d + 1 \neq 0.$$

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