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LINES ON BRIESKORN-PHAM SURFACES

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Abstract

By using toric modifications and a result of Gonzalez-Sprinberg and Lejeune-Jalabert, we answer the following questions completely. On which Brieskorn-Pham surface there exist smooth curves passing through the singular point? If there exist, how "many" and what are the defining equations?

1. Introduction

Let $(X, 0)$ be an analytic space germ embedded in $(\boldsymbol{C}^n, 0)$, and $(L, 0)$ be a smooth curve germ in $(\boldsymbol{C}^n, 0)$. Since L is locally biholomorphic to a line, we often say that L is a line. If for a singular point $O \in X_{\mathrm{sing}}$, there exists a line (smooth curve) L in $\boldsymbol{C}^n$ such that $O \in L$ and $L \setminus \{O\} \subset X_{\mathrm{reg}}$, we say that X *contains* (*or has*) *a line* passing through O.

On a singular surface X in $\boldsymbol{C}^3$ one can not always find a smooth curve passing through (not contained in) the singular locus of X. Gonzalez-Sprinberg and Lejeune-Jalabert proved a criterion for the existence of smooth curve on any (two dimensional) surface. We quote this result here for the convenience of the reader. Let $\pi : \tilde{X} \to (X, 0)$ be the minimal resolution of a singular surface $(X, 0)$. Let $\mathscr{L}$ be the set of lines on $(X, 0)$. For an exceptional divisor E, let $\mathscr{L}_E$ denote the set of lines on $(X, 0)$ with the strict transform intersecting E transversally. Let $\mathfrak{m}$ be the maximal ideal of the local ring $\mathcal{O}_{X,0}$. The cycle $\mathscr{Z}_X$ on $\tilde{X}$ defined by the ideal sheaf $\mathfrak{m}\mathcal{O}_{\tilde{X}}$ is called the *maximal cycle of* π.

GONZALEZ-SPRINBERG AND LEJEUNE-JALABERT THEOREM ([1, 2]). (1) *There exists lines on* $(X, 0)$ *if and only if the maximal cycle* $\mathscr{Z}_X$ *has at least one reduced component*;

(2) *The set of all the lines on* $(X, 0)$ *is a disjoint union of the* $\mathscr{L}_E$*'s with each* E *a reduced component in* $\mathscr{Z}_X$. *And* $\mathscr{L}_E$ *is called the family of lines corresponding to* E.

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