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INVOLUTIONS FIXING THE DISJOINT UNION OF 3-REAL PROJECTIVE SPACE WITH DOLD MANIFOLD*

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Abstract

In this paper, we determine the existence of all involutions fixing a disjoint union of 3-real projective space $RP(3)$ with Dold manifold under the condition that the normal bundle to $RP(3)$ does not bound, and also study the representatives up to bordism of those involutions which exist.

§1. Introduction

Let (T, M) be an involution on a closed manifold, and let F denote the fixed point set of (T, M). When F is chosen as

$$\{\mathrm{pt}\} \sqcup S^m, \quad RP(2k), \quad RP(m) \sqcup RP(n), \quad \sqcup RP(2l+1)(l \text{ fixed}),$$
$$\sqcup_{i=1}^{p} RP(2l_i+1), \quad \sqcup_{i=1}^{r} (S^1)^{k_i},$$

and $(S^{n_1} \times S^{n_2} \times \cdots \times S^{n_p}) \sqcup \{\mathrm{pt}\}$, respectively, the existence and the representative (up to bordism) of (T, M) have been studied in [2], [11], [9], [13], [4], [12] and [8]. The purpose of this paper is to determine the existence and the representative up to bordism of all involutions fixing a disjoint union $RP(3) \sqcup P(m, n)$, where $P(m, n)$ is the Dold manifold of dimension $m + 2n$ obtained from the product $S^m \times CP(n)$ of the m-sphere with the n-dimensional complex projective space by identifying (x, z) with $(-x, \bar{z})$ (here $(x, z) \in S^m \times CP(n)$). For this purpose, we first study the vector bundle over Dold manifold so that we can begin with our discussion on the existence of all involutions. The main method will be a formula given by Kosniowski and Stong in [5], and Lucas Theorem [10] will also be used. By setting an involution on Dold manifold $P(3, n+1)$, we partially give the representatives up to bordism of those involutions which exist.

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