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GEOMETRIC PROBABILITIES CONCERNING LARGE RANDOM TRIANGLES IN THE HYPERBOLIC PLANE

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Abstract

Concerning a random triangle on a disk D_R of radius R in the hyperbolic plane, the following four geometric probabilities are studied: (i) the probability $p_a(R)$ that a random triangle is acute; (ii) the probability $p_o(R)$ that a random triangle has the orthocenter; (iii) the probability $p_e(R)$ that a random triangle has at least one of the three excenters; and (iv) the probability $p_c(R)$ that a random triangle has the circumcenter. It is shown that, as R tends to the infinity, both the probability $p_a(R)$ and $p_o(R)$ tend to one, whereas the probability $p_e(R)$ tends to zero. Moreover it is shown that the probability $p_c(R)$ tends to a limit p_c, which can be expressed as a certain expectation concerning a random triangle in the Euclidean plane. To evaluate this expectation numerically, we obtain 0.45962039 as an estimate for p_c.

1. Introduction

The first problem concerning random triangles in the Euclidean plane is perhaps the problem "what is the probability that a random triangle is acute?". This problem was proposed by [11], and as an answer to the problem, it gave the probability $4/\pi^2 - 1/8$, assuming that three vertices of a random triangle are distributed independently and uniformly in the unit disk. Since that time various studies have been made on this problem. In [5] five different solutions to the problem are given. Whereas differences of these solutions reflect those of meanings on random triangles, they lead to the common probability $1/4$. In [10] one more solution to the problem is given, where three vertices are assumed to be distributed according as a Gaussian distribution. The answer is again $1/4$. These results are extended in [6] and [3] to the corresponding problem for random triangles in higher-dimensional Euclidean spaces.

Another problem oncerning random triangles, which has been extensively studied, is to find the distribution of the area of a random triangle whose three vertices are uniformly distributed in a given domain of Euclidean spaces. In [1], for the case that a domain is a triangle in the Euclidean plane, explicit expressions for all the moments of the area are given. In [2] these results are

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