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ON THE MULTIPLE VALUES OF ALGEBROID FUNCTIONS*

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Abstract

For any ν-valued algebroid function of finite order $\rho > 0$ in $|z| < \infty$, we prove the existence of the sequence of filling disks and Borel direction dealing with its multiple values.

1. Introduction

Valiron [1] conjectured that there exists at least a Borel direction for any ν-valued algebroid function of order ρ $(0 < \rho < \infty)$. Rauch [2] proved that there exists a direction such that the corresponding Borel exceptional values form a set of linear measure zeros. Toda [3] proved that there exists a direction such that the set of corresponding Borel exceptional values is countable. Later Lü and Gu [4] proved that there exists a direction such that the number of Borel exceptional values is equal to 2ν at most. However, it was not discussed whether there exists a Borel direction dealing with its multiple values. In the present paper we investigate this problem.

Let $w = w(z)$ be a ν-valued algebroid function in $|z| < \infty$ defined by irreducible equation

$$(1) \qquad A_\nu(z)w^\nu + A_{\nu-1}(z)w^{\nu-1} + \cdots + A_0(z) = 0,$$

where $A_\nu(z), \ldots, A_0(z)$ are entire functions without any common zero. The single valued domain of definition of $w(z)$ is a ν-sheeted covering of z-plane, a Riemann surface, denoted by $\tilde{R}_z$. A point in $\tilde{R}_z$ whose projection in the z-plane is z, is denoted by $\tilde{z}$. The part of $\tilde{R}_z$, which covers a disk $|z| < r$, is denoted by $|\tilde{z}| < r$. Let $n(r, a)$ be the number of the zeros, counted according to their multiplicities, of $w(z) - a$ in $|\tilde{z}| \le r$, $\bar{n}^{l)}(r, a)$ be the number of distinct zeros with multiplicity $\le l$ of $w(z) - a$ in $|\tilde{z}| \le r$. Let

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