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ON SECTIONAL GENUS OF QUASI-POLARIZED MANIFOLDS WITH NON-NEGATIVE KODAIRA DIMENSION, II

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Abstract

Let (X, L) be a quasi-polarized manifold over the complex number field with $\dim X = n$ and $\kappa(X) \geq 0$. If $n = 2, \kappa(X) \geq 0$, and $h^0(L) = \dim H^0(L) \geq 2$, then in our previous paper we studied a lower bound for sectional genus $g(L)$. In this paper, we mainly consider the case in which $n = 3$, $\kappa(X) \geq 0$, and $h^0(L) \geq 3$, and we obtain a lower bound for $g(L)$ which is a generalization of the result of our previous paper.

0. Introduction

Let X be a smooth projective manifold over the complex number field $\boldsymbol{C}$ with $\dim X = n$ and let L be a Cartier divisor on X. Then (X, L) is called a polarized (resp. quasi-polarized) manifold if L is ample (resp. nef and big). The sectional genus is defined by the following formula:

$$g(L) = 1 + \frac{1}{2}(K_X + (n-1)L)L^{n-1},$$

where K_X is the canonical divisor of X.

A classification of (X, L) with small value of sectional genus was obtained by several authors. On the other hand, Fujita proved the following Theorem (see Theorem (2.13.1) in [Fj0]).

THEOREM 0.1. *Let (X, L) be a polarized manifold. Then for any fixed n and $g(L)$ there are only finitely many deformation type of (X, L) unless (X, L) is a scroll over a smooth curve.*

(For a definition of the deformation type of (X, L), see §13 of Chapter II in [Fj0].) By this theorem, Fujita proposed the following Conjecture; which is interesting but difficult.

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