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KODAI. MATH. J.
23 (2000), 81–87

A CLASSIFICATION OF TRIGONAL RIEMANN SURFACES

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Abstract

Necessary and sufficient conditions are given for the existence of a trigonal Riemann surface with given genus and types of ramification points.

A compact Riemann surface W_p of genus p $(p > 3)$ is said to be *trigonal* if it admits a three sheeted covering of the Riemann sphere. The fibers of this map form a linear series g_3^1. All non-hyperelliptic Riemann surfaces of genus 4 are trigonal and generically admit two distinct g_3^1's. Exceptionally, a non-hyperelliptic W_4 admits a unique g_3^1 which is half-canonical. For $p > 4$ the locus of trigonal Riemann surfaces in moduli space has codimension $p - 4$. Also for $p > 4$ a trigonal Riemann surface has a unique g_3^1.

For a trigonal Riemann surface W_p, there is a unique integer l so that the complete linear series $|(l)g_3^1|$ and $|(l+1)g_3^1|$ have dimensions l and $l+\varepsilon$ respectively where $\varepsilon > 1$. We shall call l the *trigonal index* of W_p and denote it ti. (Then $ti \leq p/2$.)

Now we consider the ramification points of the three sheeted cover using the terminology introduced by Coppens. Ramification points of multiplicity two will be called *ordinary*, and those of multiplicity three will be called *total*. Ramification points occurring in the residual divisor (defined below), when $ti < p/2$, will be said to be of Type II, and otherwise of Type I. If $ti = p/2$ then all ramification points are of Type I. Thus there are four kinds of ramification points.

Recently trigonal Riemann surfaces have been studied extensively in the context of the trigonal-tetragonal relation, but from the viewpoint of this paper the literature in more sparse. Despite the work of Hensel-Landsberg [4] concerning the dimension in moduli space of the loci of trigonal surfaces with given trigonal index, very little seems to have occurred until Coppens distinguished the four kinds of ramification points [2], [3]. Then Kato and Horiuchi [6], [5] derived a canonical algebraic equation of degree three defining a trigonal Riemann surface. This derivation for trigonal surfaces occurred more than a century after the corresponding derivation for hyperelliptic surfaces. It appears that the work

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1991 *Mathematics Subject Classification*: 14H.

Received March 8, 1999; revised June 30, 1999.