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HITTING DISTRIBUTION TO A QUADRANT OF TWO-DIMENSIONAL RANDOM WALK

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Let $H_L(\xi,\eta)$ be the probability that a two-dimensional simple random walk starting at ξ hits the third quadrant L for the first time at η. The main objective of this paper is to investigate the asymptotic behavior of $H_L(\xi,\eta)$. It is especially proved that there exists a constant C_0 such that for $\xi \in \mathbf{Z}^2\backslash L$ and $l \in \mathbf{N}$,

$$|H_L(\xi,(-l,0)) - h_L(\xi,(-l,0))| \le C_0\{|\xi + (l,0)|^{-3} + |\xi|^{-2/3}l^{-5/3}\},$$

where $h_L(\xi,\cdot)$ is the density of the hitting distribution to the third quadrant of two-dimensional standard Brownian motion starting at ξ. This estimate is sharp at least in the sense that the powers $-2/3$ and $-5/3$ can not be improved.

1. Introduction and statements of results

Let $\{S(n)\}_{n=0}^{\infty}$ be a two-dimensional simple random walk starting at $\xi \in \mathbf{Z}^2$, namely,

$$S(0) = \xi \quad \text{and} \quad S(n) = S(0) + \sum_{k=1}^{n} X_k,$$

where $X_1, X_2, \ldots$ is a sequence of independent, identically distributed random variables that take four values $(1,0), (-1,0), (0,1), (0,-1)$ with equal probability. We denote by P_ξ the probability law of the process $\{S(n)\}_{n=0}^{\infty}$. For a subset A of $\mathbf{R}^2$ such that $A \cap \mathbf{Z}^2 \neq \emptyset$, define

$$\tau_A = \inf\{n \ge 1 : S(n) \in A\},$$

the hitting time of A. Since $S(n)$ is recurrent, $\tau_A < \infty$ a.s. The hitting distribution $H_A(\xi,\eta)$ is defined by

$$H_A(\xi,\eta) = P_\xi\{S(\tau_A) = \eta\}, \quad (\xi \in \mathbf{Z}^2\backslash A, \eta \in A \cap \mathbf{Z}^2).$$

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