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UNIQUENESS OF ENTIRE FUNCTIONS THAT SHARE SOME SMALL FUNCTIONS

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Abstract

In this paper we obtain a unicity theorem of an entire function and its derivative that share two small functions IM. So we generalize and improve some results given by Rubel–Yang, Mues–Steinmetz and J. H. Zheng etc.

1. Introduction and main results

In this paper, we use the same signs as given in Nevanlinna theory of meromorphic functions (see [1]). By $S(r,f)$ we denote any quantity satisfying $S(r,f)=o\{T(r,f)\}$ as $r\to\infty$, possibly outside a set of r with finite linear measure. Let f and g be two meromorphic functions. Then the meromorphic function α is said a small function of f if and only if $T(r,\alpha)=S(r,f)$. We say that f and g share a value a IM(CM) if $f-a$ and $g-a$ have the same zeros ignoring multiplicities (with the same multiplicity). When a is a small function of f and g, a is said a common small function of f and g IM(CM). In addition, we introduce the following denotations:

$S(m,n)(b)=\{z|z$ is a common zero of $f-b$ and $f'-b$ with multiplicities m and n respectively$\}$. $\bar{N}(m,n)(r,1/(f-b))$ denotes the counting function of f with respect to the set $S(m,n)(b)$.

On the problems of uniqueness of an entire function and its derivative that share some values, Rubel–Yang (see [2]) proved that if the entire function f and f' share two distinct finite values CM then $f\equiv f'$. Mues–Steinmetz (see [3]) improved this result to the case when f and f' share two values IM. In 1992, J. H. Zheng and S. P. Wang (see [4]) generalized this result to the f and f' which share two small functions CM. In this paper, we generalize and improve above results to obtain the following result:

THEOREM 1. *Let f be a nonconstant entire function, a and b two distinct small functions of f with $a\not\equiv\infty$ and $b\not\equiv\infty$. If f and f' share a and b IM, then $f\equiv f'$.*

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