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EULER SYSTEMS, IWASAWA THEORY, AND SELMER GROUPS

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Introduction

Kolyvagin discovered the method of Euler system, and used it to analyze ideal class groups of certain cyclotomic fields and Selmer groups of elliptic curves. Rubin used the method of Euler system to obtain a new proof of Iwasawa main conjecture and a proof of the main conjecture for imaginary quadratic fields ([Ko_1], [Ko_2], [Ru_1], [Ru_2]). It is vaguely believed that once a nice Euler system is discovered, we can analyze certain étale cohomology groups and "Selmer groups" which are generalizations of ideal class groups and of Selmer groups of elliptic curves. This paper is an attempt to prove the truth of this belief. In this paper, we show that once a nice Euler system of a p-adic representation of $\mathrm{Gal}(\bar{\boldsymbol{Q}}/\boldsymbol{Q})$ is given (see Proposition 1.1 for the meaning of "an Euler system for a p-adic representation of $\mathrm{Gal}(\bar{\boldsymbol{Q}}/\boldsymbol{Q})$"), then we can prove finiteness theorems for the second étale cohomology $H^2_{\text{ét}}$ (Theorem 13.3) and for the Selmer group (Theorem 13.2) of the Galois representation, and can prove a part of an analogue of Iwasawa main conjecture (Theorem 0.8) of the Galois representation.

During I was preparing this paper, I leaned that similar results were obtained also by B. Perrin-Riou and by K. Rubin, independently ([Pe], [Ru_4]). The results of this paper will be used in [Ka_2] to develop the Iwasawa theory of elliptic cusp forms and Iwasawa theory of elliptic curves without complex multiplication. Results of this paper on $H^2_{\text{ét}}$ and Selmer groups are obtained under the assumption that we are given a nice Euler system, and how to find an Euler system is a difficult problem. In [Ka_2], we actually find nice Euler systems for two dimensional Galois representations associated to elliptic cusp forms. These Euler systems come from Beilinson's elements in K_2 of modular curves ([Be]).

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§0. Main result

In this §0, we fix the meaning of "an Euler system for p-adic representation of $\mathrm{Gal}(\bar{\boldsymbol{Q}}/\boldsymbol{Q})$" (cf. 0.1) in our sense, and state the main result Theorem 0.8 of this

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