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GEOMETRY OF F-HARMONIC MAPS

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1. Introduction

Harmonic maps are critical points of the energy functional defined on the space of smooth maps between Riemannian manifolds. There are many studies on harmonic maps. Also, p-harmonic maps and exponentially harmonic maps have been developed. Baird and Eells [BE], and Takeuchi [T] studied some conformal properties of harmonic maps and p-harmonic maps, respectively. They showed that if the dimension of the target manifold is equal to 2 (resp. p), then the fibers of harmonic morphisms (resp. horizontally conformal p-harmonic maps) are minimal submanifolds in the domain manifold. Leung [L], Cheung and Leung [CL], and Koh [K] discussed the stability of harmonic maps, p-harmonic maps and exponentially harmonic maps, respectively.

We would like to construct a unified theory for several varieties of harmonic map. We give the notion of F-harmonic maps, which is a generalization of harmonic maps, p-harmonic maps or exponentially harmonic maps.

In this paper, we discuss some conformal properties and the stability of F-harmonic maps. Our results are extensions of [BE], [T] for conformal properties, and [L], [CL], [K] for the stability. We can see results for harmonic maps, p-harmonic maps or exponentially harmonic maps in a different viewpoint.

Let $F : [0,\infty) \to [0,\infty)$ be a C^2 function such that $F' > 0$ on $(0,\infty)$. For a smooth map $\phi : (M,g) \to (N,h)$ between Riemannian manifolds (M,g) and (N,h), we define the F-energy $E_F(\phi)$ of ϕ by

$$E_F(\phi) = \int_M F\left(\frac{|d\phi|^2}{2}\right) v_g,$$

where $|d\phi|$ denotes the Hilbert-Schmidt norm of the differential $d\phi \in \Gamma(T^*M \otimes \phi^{-1}TN)$ with respect to g and h, and v_g is the volume element of (M,g). It is the energy, the p-energy, the α-energy of Sacks-Uhlenbeck [SU] and the exponential energy when $F(t) = t, (2t)^{p/2}/p$ $(p \geq 4)$, $(1+2t)^\alpha$ $(\alpha > 1$, $\dim M = 2)$ and e^t, respectively. We shall say that ϕ is an F-harmonic map if it is a critical point of the F-energy functional, which is a generalization of harmonic maps, p-harmonic maps or exponentially harmonic maps.

This paper is organized as follows. In Section 2, we derive the first variation formula for F-harmonic maps, and have a certain relation between F-harmonic maps and harmonic maps through conformal deformations. In