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LOG BETTI COHOMOLOGY, LOG ÉTALE COHOMOLOGY, AND LOG DE RHAM COHOMOLOGY OF LOG SCHEMES OVER C

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§0. Introduction

The purpose of this paper is to extend the classical relationship between Betti cohomology, étale cohomology and de Rham cohomology for varieties over the complex number field $\boldsymbol{C}$ to the logarithmic geometry over $\boldsymbol{C}$ in the sense of Fontaine-Illusie.

We state the main results Theorem (0.2) and Theorem (0.5) of this paper.

(0.1). For varieties over $\boldsymbol{C}$, the above three cohomology theories are closely related. We have:

(1) **(étale vs. Betti)** Let X be a scheme locally of finite type over $\boldsymbol{C}$, and let F be a constructible sheaf of (torsion) abelian groups on the étale site $X_{\text{ét}}$. Then, we have

$$H^q(X_{\text{ét}}, F) \cong H^q(X_{\text{an}}, F_{\text{an}}) \quad \text{for any } q \in \boldsymbol{Z},$$

where X_{an} is the analytic space associated to X and F_{an} is the inverse image of F on X_{an}. (Cf. [AGrV] XVI 4.1. See also the proof of Theorem (2.6).)

(2) **(de Rham vs. Betti)** Let X be a smooth scheme over $\boldsymbol{C}$. Then, we have

$$\boldsymbol{H}^q(X, \Omega^{\cdot}_{X/\boldsymbol{C}}) \cong H^q(X_{\text{an}}, \boldsymbol{C}) \quad \text{for any } q \in \boldsymbol{Z},$$

where $\Omega^{\cdot}_{X/\boldsymbol{C}}$ is the de Rham complex of X ([Gr2]).

In this paper, we prove generalizations of these results to schemes over $\boldsymbol{C}$ endowed with logarithmic structures in the sense of Fontaine-Illusie.

Let X be an fs log scheme ([N1] (1.7)) over $\boldsymbol{C}$ whose underlying scheme $\mathring{X}$ is locally of finite type over $\boldsymbol{C}$. Then the analytic space X_{an} associated to $\mathring{X}$ is endowed with the inverse image of the log structure of X. For an analytic space Y over $\boldsymbol{C}$ endowed with an fs log structure (like X_{an}), we will define a topological space $Y^{\log}$ which is endowed with a continuous surjective map $\tau : Y^{\log} \to Y$ ((1.2)). We denote $(X_{\text{an}})^{\log}$ by $X_{\text{an}}^{\log}$. We prove:

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