H. KOJIMA
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REMARK ON FOURIER COEFFICIENTS OF MODULAR FORMS OF HALF INTEGRAL WEIGHT BELONGING TO KOHNEN'S SPACES II

HISASHI KOJIMA

Introduction

In [13], Waldspurger first found that the square of Fourier coefficients $a(n)$ at a square free integer n of modular forms $f(z) = \sum_{n=1}^{\infty} a(n)e[nz]$ of half integral weight is essentially proportional to the central value of the zeta function at a certain integer attached to the modular form F if f corresponds to F by the Shimura correspondence Ψ and f is an eigen-function of Hecke operators. Kohnen-Zagier [1], [3] determined explicitly the constant of the proportionality in the case of modular forms belonging to Kohnen's spaces $S_{(2k+1)/2}(N,\chi)$ of weight $(2k+1)/2$ and of square free level N with character χ which is a subspace of $S_{(2k+1)/2}(4N,\chi_1)$, where $S_{(2k+1)/2}(4N,\chi_1)$ means the space of modular cusp forms of half integral weight given in [9]. Kohnen-Zagier [1] (resp. Kohnen [3]) treated the case where $N = 1$ (resp. N is an odd square free integer and χ is the trivial character of level N) (cf. Kojima [4] and [5]).

In [12], Shimura intended to generalize such formulas to the case of Hilbert modular forms f of half integral weight and succeeded in obtaining many general interesting formulas. Among these, some explicit and useful formulas about the proportionality constant were formulated under assumptions that f satisfies the multiplicity one theorem. For modular forms belonging to Kohnen's spaces $S_{(2k+1)/2}(N,\chi)$, Shimura [12] did not give the same explicit formula as that of Kohnen and Zagier [1], [3].

In [6], we represented explicitly the ratio of the square of Fourier coefficients $a(n)$ at a fundamental discriminant n of modular forms $f(z) = \sum_{\varepsilon(-1)^k n\equiv 0,1(4),n>0} a(n)e[nz]$ of weight $(2k+1)/2$ belonging to the Kohnen's space $S_{(2k+1)/2}(N,\chi)$ by the central value of the zeta function of the modular form F which is the image of f under the Shimura correspondence Ψ, where χ is any primitive character modulo N and n is equal to 4τ with a square free integer τ satisfying $\tau \equiv 2,3 \pmod 4$.

The purpose of this paper is to remove these conditions about χ and n and to

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