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AMPLE VECTOR BUNDLES AND DEL PEZZO MANIFOLDS

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Abstract

Let $\mathscr{E}$ be an ample vector bundle of rank r on a smooth complex projective manifold X of dimension $n \geq r+3$. Pairs $(X, \mathscr{E})$ as above are investigated under the assumption that $\mathscr{E}$ has a regular section vanishing along a Fano manifold Z of index $\dim Z - 1$ and Picard number $\rho(Z) \geq 2$.

Introduction

Let X be a complex projective manifold of dimension n and let $\mathscr{E}$ be an ample vector bundle of rank $r \leq n-2$ on X having a regular section, i.e. there exists a section $s \in \Gamma(\mathscr{E})$ whose zero locus $Z := (s)_0$ is a smooth subvariety of the expected dimension $n-r$. Triplets $(X, \mathscr{E}, Z)$ as above have been investigated in several papers ([LM1], [LM2], [LM3], [dF], [LM4]) under the assumption that Z is some special variety. In particular the case when Z is a Fano manifold of index $\dim Z - 1$ and Picard number $\rho(Z) = 1$ was discussed in [LM1, (2.4)]. In this paper we focus on the case $\rho(Z) > 1$, assuming that $\dim Z \geq 3$. Actually as the results in [LPS] show, the same study when Z is a surface is far from being complete even in the case of divisors, i.e. when $r = 1$. We recall that extending several classification results known in the setting of ample divisors is the main motivation for investigating triplets $(X, \mathscr{E}, Z)$ as above [LM1].

To relate our Z to the title note that Fano manifolds of index $\dim Z - 1$ coincide with del Pezzo manifolds, with the only exception given by the pair $(\boldsymbol{P}^3, \mathcal{O}_{\boldsymbol{P}^3}(2))$. So, having assumed that $\dim Z \geq 3$, according to the classification of del Pezzo manifolds, [F, Chapter I, §8], Z is one of the following:

$$(0.1) \qquad \boldsymbol{P}^2 \times \boldsymbol{P}^2, \quad \boldsymbol{P}(T_{\boldsymbol{P}^2}), \quad \boldsymbol{B}_q(\boldsymbol{P}^3), \quad \boldsymbol{P}^1 \times \boldsymbol{P}^1 \times \boldsymbol{P}^1,$$

where $\boldsymbol{B}_q(\boldsymbol{P}^3)$ stands for $\boldsymbol{P}^3$ blown-up at a point q. Note that this threefold has

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