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NON-LINEARIZABILITY OF POLYNOMIALS AT IRRATIONALLY INDIFFERENT FIXED POINTS

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Abstract

In this paper, we consider the non-linearizability of polynomials with irrationally indifferent fixed points. Under the assumption that there exists a cubic polynomial which is linearizable at an irrationally indifferent fixed point with a non-Brjuno multiplier, we show that, for every degree more than two, one can construct a holomorphic family of possible maximal dimension consisting of polynomials linearizable at the fixed point.

1. Introduction

Let f be a germ of a holomorphic map at $z_0 \in \boldsymbol{C}$ with $f(z_0) = z_0$ and call $\lambda := f'(z_0)$ the multiplier of f at z_0. We consider the linearization problem of f at $z = z_0$, i.e. whether there exists a holomorphic local change of coordinate $z = h(w)$ with $h(0) = z_0$ and $h'(0) \neq 0$ which conjugates f to the linear map $w \mapsto \lambda w$. If such h exists, the germ f is said to be *linearizable* at z_0 and we call h the (analytic) linearizing map of f at z_0 or the solution of the linearization problem of f at z_0.

If $\lambda = 0$, Böttcher showed that $f(z) = z^n + a_{n+1}z^{n+1} + \cdots$ is always analytically conjugate to $w \mapsto w^n$. In the case $0 < |\lambda| < 1$ (resp. $1 < |\lambda|$), Kœnigs showed that f is always linearizable at z_0 and the fixed point z_0 is called attracting (resp. repelling). If $|\lambda| = 1$ and λ is a root of unity, f is always non-linearizable at z_0 and z_0 is called parabolic (for the details, see [5]).

If $|\lambda| = 1$ and λ is not a root of unity, the fixed point z_0 of f is said to be *irrationally indifferent*. In this case, some are linearizable at z_0, others non-linearizable at z_0. For example, a rational or entire function f which has an irrationally indifferent fixed point z_0 is linearizable there if and only if the fixed point z_0 belongs to the Fatou set of f (cf. [5] and [8]).

From now on, we always assume that a real number α is irrational. Let

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