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BRAID MONODROMY OF COMPLEX LINE ARRANGEMENTS

Dedicated to the memory of Professor N. Sasakura

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Abstract

Let V be the complex vector space C^l, $\mathscr{A}$ an arrangement in V, i.e. a finite family of hyperplanes in V In [11], Moishezon associated to any algebraic plane curve $\mathscr{C}$ of degree n a braid monodromy homomorphism θ $F_s \to B(n)$, where F_s is a free group, $B(n)$ is the Artin braid group. In this paper, we will determine the braid monodromy for the case when $\mathscr{C}$ is an arrangement $\mathscr{A}$ of complex lines in C^2, using the notion of labyrinth of an arrangement. As a corollary we get the braid monodromy presentation for the fundamental group of the complement to the arrangement.

1. Introduction

Let $\mathscr{C} = \{f(x,y) = 0\} \in C^2$ be a plane algebraic curve. From the 1930's, it is well known (see [9], [17]) that the fundamental group of the complement to $\mathscr{C}$, $\pi_1(C^2 \backslash \mathscr{C})$, can be computed using the van Kampen's method. In [11], Moishezon introduced the notion of braid monodromy of $\mathscr{C}$. Suppose that the projection on the x-axis, $pr_1 : C^2 \to C^1$, is generic with respect to the curve $\mathscr{C}$. Let $S(\mathscr{C}) = \{\alpha \in \mathscr{C}; \partial f(\alpha)/\partial y = 0\}$ and $D(\mathscr{C})$ its image under pr_1. Then the braid monodromy of $\mathscr{C}$ is a homeomorphism $\theta : \pi_1(C^1 \backslash D(\mathscr{C})) \to B[pr_1^{-1}(x_0), pr_1^{-1}(x_0) \cap \mathscr{C}]$, where $x_0 \in C^1 \backslash D(\mathscr{C})$ is a base point.

An arrangement $\mathscr{A}$ is a finite family of hyperplanes in C^l. Given an arrangement $\mathscr{A}$, an algorithm to compute the fundamental group of the complement, $\pi_1(C^l \backslash \bigcup_{H \in \mathscr{A}} H)$, was proved in [14] when $\mathscr{A}$ is the complexification of a real arrangement. Similar results were obtained in [5] and [16] by different methods. For an arbitrary complex arrangement a standard argument using the Zariski hyperplane section theorem (see e.g. [7]) reduces the problem to the case when $\mathscr{A}$ is an arrangement of complex lines in C^2. Arvola [1] found an

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