B. SRIVASTAVA
KODAI MATH. J.
22 (1999), 35–45

ON THE BEHAVIOUR OF MOCK θ-FUNCTIONS FOUND IN THE "LOST" NOTE BOOK

BHASKAR SRIVASTAVA

1. Introduction

From Ramanujan's 'lost' note book Andrews and Hickerson [1] stated the seven mock theta functions $\phi(q)$, $\psi(q)$, $\rho(q)$, $\sigma(q)$, $\lambda(q)$, $\mu(q)$ and $\nu(q)$. In their paper Andrews and Hickerson [1] while studying the identities connecting these seven functions did not actually find their asymptotic behaviour in the neighbourhood of a point of a unit circle as $|q| \to 1$. Earlier Watson [6, 7] and Dragonette [3] had discussed in some detail the behaviour of the third and fifth order mock theta functions. The asymptotic behaviour of the seventh order mock theta functions has been discussed by Selberg [5]. The object of this paper is to study in detail the asymptotic behaviour of the seven mock theta functions found in the 'lost' notebook, in their 'bilateral' forms as defined in the next section below.

2. Notation and definitions

If $n \geq 0$, we define

$$(x)_n = (x;q)_n = \prod_{\iota=0}^{n-1}(1 - q^\iota x).$$

If $|q| < 1$, we let

$$(x)_\infty = (x;q)_\infty = \lim_{n\to\infty}(x)_n = \prod_{\iota \geq 0}(1 - q^\iota x)$$

and more generally

$$\begin{aligned}(x_1, \ldots, x_r; q)_\infty &= (x_1)_\infty \cdots (x_r)_\infty \\ &= \prod_{\iota \geq 0}(1 - q^\iota x_1) \cdots (1 - q^\iota x_r).\end{aligned}$$

Received February 12, 1998.