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CIRCLES IN RIEMANNIAN SYMMETRIC SPACES

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Abstract

We show that every circle in a compact Riemannian symmetric space of rank one is obtained as an orbit of a one parameter subgroup of isometries. We also show that a homogeneous space with the above property is either a Euclidean space or a Riemannian globally symmetric space of rank one.

Introduction

Let (M, g) be a Riemannian manifold and ∇ the Riemannian connection of (M, g). An arc-length parametrized curve $c(t)$ in (M, g) is called a *circle* if there exist a unit vector field $Y(t)$ along $c(t)$ and a positive constant k such that

$$\nabla_{\dot{c}(t)}\dot{c}(t) = kY(t), \quad \nabla_{\dot{c}(t)}Y(t) = -k\dot{c}(t).$$

The constant k is called the *curvature* of the circle. Let $\{X, Y\}$ be an arbitrary pair of mutually orthogonal unit vectors in T_pM at a point $p \in M$ and k be a positive constant. There exists a unique circle $c : (-\varepsilon, \varepsilon) \to M$ with the initial condition

$$c(0) = p, \quad \dot{c}(0) = X, \quad (\nabla_{\dot{c}}\dot{c})(0) = kY$$

for sufficiently small ε. It is known that in a complete Riemannian manifold every circle can be defined for $-\infty < t < \infty$.

Recently Adachi, Maeda and Udagawa [3] studied the circles in a complex projective space $P^n(\boldsymbol{C})$ of constant holomorphic sectional curvature. For instance, they proved that a circle in $P^n(\boldsymbol{C})$ is characterized by the curvature k and the complex torsion. Adachi [1] studied the similar problem for a quaternion projective space and its non-compact dual. Adachi and Maeda also studied circles in complex hyperbolic space [2]. One of the purpose of this paper is to generalize their results to circles in compact Riemannian symmetric spaces of rank one.

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