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MINIMAL H_3 ACTIONS AND SIMPLE QUOTIENTS OF A DISCRETE 6-DIMENSIONAL NILPOTENT GROUP

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Abstract

A simple C^*-algebra is introduced that is generated by a minimal effective action of the (discrete, nilpotent, non-abelian) Heisenberg group H_3 on the torus T^2 It appears as a simple quotient of the group C^*-algebra $C^*(H_{6,4})$ of a 6-dimensional discrete nilpotent group $H_{6,4}$, and also as a C^*-crossed product generated by an action of Z^2 on T^3 The rest of the infinite dimensional simple quotients of $C^*(H_{6,4})$ are identified and displayed as C^*-crossed products generated by minimal effective actions, and also as matrix algebras over simple C^*-algebras from groups of lower dimension.

1. Introduction

In 3 dimensions there is a unique (up to isomorphism) connected, simply connected, nilpotent Lie group, which we call G_3 (following Nielsen [N]); G_3 ($= \boldsymbol{R}^3$ as a set) is the Heisenberg group with multiplication

$$(k,m,n)(k',m',n') = (k+k'+nm', m+m', n+n').$$

The faithful irreducible representations of the lattice subgroup H_3 ($= \boldsymbol{Z}^3$ as a set) of G_3 generate the irrational rotation algebras A_θ. In 4 dimensions there is also a unique such connected group G_4, and in 5 dimensions there are 6 such groups $G_{5,i}$, $1 \leq i \leq 6$. The main thrust in [MW1, MW2] was to find cocompact subgroups $H_4 \subset G_4$ and $H_{5,i} \subset G_{5,i}$, that would be analogous to $H_3 \subset G_3$, and then for these H's to identify the infinite dimensional simple quotients of $C^*(H)$, both the faithful ones (generated by a faithful representation of H) and the non-faithful ones, and also to give matrix representations over lower dimensional algebras for as many of the non-faithful quotients as possible. In the course of this work, it was observed that all flow presentations of simple C^*-algebras that arose used actions of abelian groups, namely, $\boldsymbol{Z}$ or $\boldsymbol{Z}^2$, or subgroups of them. It was also observed in the 5-dimensional setting that one of the groups, $H_{5,6}$,

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