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KODAI MATH. J.
20 (1997), 269—274

ON THE ŁOJASIEWICZ EXPONENT AT INFINITY FOR POLYNOMIAL FUNCTIONS

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1. Introudction

1.1. For $n, q \in \boldsymbol{N} \setminus \{0\}$ we consider the polynomial functions

$$f = f_{n,q} : \boldsymbol{C}^3 \longrightarrow \boldsymbol{C}, \quad f(x, y, z) = f_{n,q}(x, y, z) := x - 3x^{2n+1}y^{2q} + 2x^{3n+1}y^{3q} + yz.$$

We will study some properties of these polynomials, related to their behaviour at infinity, and we will prove that some results, obtained in [6] and [3], [4], [7] for the case of polynomials in two variables, are not true in the case of polynomials in $m \geqq 3$ variables. Also, our polynomials $f_{n,q}$ show that several classes of polynomials, with "good" behaviour at infinity, considered in [8], [9], [14], [10], are distinct.

The first remark on our polynomials is:

1.2. *Remark.* After a suitable polynomial change of coordinates in $\boldsymbol{C}^3$, one can write $f(X, y, Z) = X$. Namely, taking $Z := z - 3x^{2n+1}y^{2q-1} + 2x^{3n+1}y^{3q-1}$, we get: $f(x, y, Z) = x + yZ$. Next, we put $X := x + yZ$ and we obtain $f(X, y, Z) = X$. Thus, there exists a polynomial automorphism $P = (P_1, P_2, P_3) : \boldsymbol{C}^3 \rightarrow \boldsymbol{C}^3$ such that $f = P_1$.

1.3. For a polynomial $g : \boldsymbol{C}^m \rightarrow \boldsymbol{C}$, we consider grad $g(x) := (\overline{\partial f / \partial x_1(x)}, \ldots, \overline{\partial f / \partial x_m(x)})$. If g has non-isolated singularities, the *Łojasiewicz number at infinity*, $L_\infty(g)$, is defined by $L_\infty(g) := -\infty$. When g has only isolated singularities, the Łojasiewicz number at infinity is the supremum of the set

$$\{\nu \in \boldsymbol{R} \mid \exists A > 0, \exists B > 0, \forall x \in \boldsymbol{C}^m, \text{ if } \|x\| \geqq B, \text{ then } A\|x\|^\nu \leqq \|\operatorname{grad} g(x)\|\}.$$

Equivalent definition is (see for instance [6] or [5], proof of Proposition 1):

$$L_\infty(g) := \lim_{r \to \infty} \frac{\log \varphi(r)}{\log r}, \quad \text{where } \varphi(r) := \inf_{\|x\| = r} \|\operatorname{grad} g(x)\|.$$

The following result is a reformulation of Theorem 10.2 from [4]:

Received July 2, 1997.