M. MUKAI
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THE DEFORMATION OF HARMONIC MAPS GIVEN BY THE CLIFFORD TORI

MARIKO MUKAI

Introduction

The purpose of this paper is to provide some new results on deformations for harmonic maps. Let ϕ be a harmonic map of a compact Riemannian manifold M into a Riemannian manifold N. A one-parameter family $\phi(t)$ of harmonic maps such that $\phi(0)=\phi$ is called a *harmonic deformation of* ϕ. Then each $\phi(t)$ satisfies the harmonic map equations:

$$\tau(\phi(t))\equiv 0, \tag{0.1}$$

where $\tau(\phi)$ denotes the tension field of ϕ. By taking a derivative of the equation (0.1) at $t=0$, we have the equation

$$\frac{d}{dt}\tau(\phi(t))\Big|_{t=0}\equiv \mathcal{J}_{\phi}(\dot{\phi})=0, \quad \dot{\phi}\in C^{\infty}(\phi^{-1}TN). \tag{0.2}$$

Here $\mathcal{J}_{\phi}$ denotes the Jacobi operator of the energy functional. If a section $v\in C^{\infty}(\phi^{-1}TN)$ of $\phi^{-1}TN$ satisfies the equation (0.2), then it is called an *infinitesimal harmonic deformation* (or a *harmonic i-deformation*) *of* ϕ. We denote by HID(ϕ) the vector space of all harmonic i-deformations of ϕ. The space HID(ϕ) just coincides with the vector space Ker$\mathcal{J}_{\phi}$ of all Jacobi fields of ϕ. If $v\in$HID(ϕ) generates harmonic deformations, then v is said to be *integrable*. Let Harm(M, N) denote the space of all harmonic maps of M into N. From the point of view of the deformation theory of harmonic maps, the following are fundamental problems;

(1) to ask whether or not all harmonic i-deformations of ϕ are integrable,
(2) to make its cause clear if an harmonic i-deformation which is not integrable appears,
(3) to investigate the structure of a neighborhood in Harm(M, N) around ϕ,
(4) to determine the connected component in Harm(M, N) containing ϕ and to examine its compactness, if it is noncompact, to construct its natural compactification.

Because of the finiteness of the dimension of Ker$\mathcal{J}_{\phi}$, we know that Harm(M,

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