

## COMPLETE MAXIMAL SPACELIKE SUBMANIFOLDS

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### Abstract

We generalize Simons' method to spacelike submanifolds of  $M_q^{n+p}(c)$  ( $1 \leq q \leq p$ ) and characterize the totally geodesic submanifolds of  $S_q^{n+p}(c)$  ( $1 \leq q \leq p$ ) under the pinching conditions on scalar curvature, Ricci curvature and sectional curvature, respectively.

### 1. Introduction

Let  $M_q^{n+p}(c)$  be an  $(n+p)$ -dimensional connected indefinite Riemannian manifold of index  $q$  ( $1 \leq q \leq p$ ) and of constant curvature  $c$ , which is called an indefinite space form of index  $q$ . According to  $c > 0$ ,  $c = 0$  and  $c < 0$ , it is denoted by  $S_q^{n+p}(c)$ ,  $R_q^{n+p}$  or  $H_q^{n+p}(c)$ . A submanifold  $M^n$  of an indefinite space form  $M_q^{n+p}(c)$  is said to be *spacelike* if the induced metric on  $M^n$  from that of  $M_q^{n+p}(c)$  is positive definite.  $R^n$  can be embedded in  $S_1^{n+1}(c)$  as a complete totally umbilical spacelike submanifold. But it can not be embedded in the unit sphere  $S^m(c)$  as a totally umbilical submanifold. Hence it is very interesting to investigate complete spacelike submanifolds in  $M_q^{n+p}(c)$ .

When  $p = q$ , we know that complete maximal spacelike submanifolds in  $S_p^{n+p}(c)$  or  $R_p^{n+p}$  are totally geodesic (cf. [3]). Hence the class of all such submanifolds are very small. But if  $q < p$  we shall see that the class of complete maximal spacelike submanifolds is very large. In fact, if  $M^n$  is a complete minimal submanifold in sphere  $S^m(c)$  ( $m > n$ ) of constant curvature  $c$  embedded in  $S_q^{m+q}(c)$  as a totally geodesic spacelike submanifold where  $m - n + q = p$ , then  $M^n$  is a complete maximal spacelike submanifold in  $S_q^{n+p}(c)$ . In [1], Alias and Romero studied the compact maximal spacelike submanifolds in  $S_q^{n+p}(c)$ . They proved that if  $M^n$  is a compact maximal spacelike submanifold in  $S_q^{n+p}(c)$  with Ricci curvature  $\text{Ric}(M^n) \geq (n-1)c$ , then  $M^n$  is totally geodesic. And they indicated that to get a Bernstein type result, the bound on the Ricci curvature

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